**586.**

## Problem 42.28 (RHK)

***The twin paradox***. *Consider a pair of twins Fred and Ethel each having identical clocks that were previously calibrated to keep Earth time. The clock can be used, in their respective frames, to record the passage of time in that frame in units of Earth years* (*but of course the years appear to be of different durations if the frames are in relative motion*). *Let us suppose that Ethel in her spaceship is moving away from Fred and his spaceship at a star stationary with respect to Fred and whose distance from the platform the platform is measured by Fred to be* 12 *light years. Ethel arrives at the star with speed* $u = 0.6c$ *and reverses her journey and travels with the same speed* $0.6c$ *back to Fred and compare their ages. After the return trip Ethel finds that she is younger than Fred. We will analyse the problem from the perspective of Fred as well that of Ethel using the Doppler shift analysis.*

**Solution:**

What is called the "*twin paradox*" is only an apparent paradox. In the common formulation of the paradox the fact that the twins are not symmetrically situated is not pointed out till a paradoxical outcome is obtained. While one of the twins remains in an inertial frame in which the distant star is also at rest, the other twin has to necessarily undergo accelerations during its onward and return journeys performed apparently with constant speeds. Therefore, the frames of reference of one of the twins are noninertial, whereas the other twin continues to be in an inertial frame while the moving twin makes a round trip journey.

We will analyse the perspectives of Fred (the inertial twin) and that of Ethel (the noninertial twin) by applying relativistic Doppler shift.

According to Fred the time taken by Ethel in her journey to the star, which is 12.6 light-year away, at a speed of 0.6*c*, would be $20\ \text{years}\ (12.6\ \text{light-year}/0.6c)$. The same amount of time would have been measured by Fred for Ethel's return journey, as she travels with the same speed 0.6*c*.

From the perspective of Ethel the star is moving toward her with speed 0.6*c*. The lapse of time during which the star crosses her as per her clocks would be $20\times\sqrt{1-0.6^2}$ years $=16$ years. Her clocks would also show lapse of 16 years when she returns to Fred from the star travelling at speed of 0.6*c* with respect to Fred. Therefore, Fred would have aged by 40 years while Ethel would have aged by 32 years as per their respective clocks when they meet after Ethel's return trip. This appears as a paradoxical situation, as long as the rest frame of Ethel and the rest frame of Fred are considered as interchangeable. This is not correct, because Fred continues to remain in an inertial frame while Ethel's rest frames are noninertial.

We will analyse this situation by first assuming that Fred sends light signals each year on his birthday to Ethel. In the second part we will analyse the situation by assuming that Ethel sends light signals to Fred each year on her birthday.

During the round trip of Ethel's journey Fred would have sent 40 light signals to her, as the frequency of signals sent by Fred is 1 per year. During her outward journey the frequency of signals received by Ethel would be given by the Doppler shift formula to be

$$\nu = \left(1\ \text{y}^{-1}\right) \times \sqrt{\frac{1-0.6}{1+0.6}} = 0.5\ \text{y}^{-1}\ .$$

That is Ethel would measure that light signals received from Fred have frequency of one every 2 years of her clock. Therefore, during the 16 years of her clock, when she and Fred were moving away from each other with speed of 0.6*c*, she would have received 8 signals from Fred, and according to her, as per their previous arrangement, she would conclude that while she has celebrated 16 birthdays Fred would have celebrated only 8 birthdays.

During her return trip she would measure that frequency of the signals sent by Fred would be

$$\nu' = \left(1\ \text{y}^{-1}\right) \times \sqrt{\frac{1+0.6}{1-0.6}} = 2\ \text{y}^{-1}.$$

That is 2 Ethel would receive two signals per year from Fred. Therefore, during her return trip she would have received 32 signals from Fred. Ethel would conclude that during her round trip while her clocks show 32 years Fred would have aged by 40 years.

We will next analyse the problem from Fred's perspective, assuming that Ethel sends 32 light signal one on each of her birthdays. During Ethel's onward journey, as Fred and Ethel are moving away from each other with relative speed 0.6*c*, the frequency of signals received by Fred would be

$$\nu = \left(1\ \mathrm{y}^{-1}\right) \times \sqrt{\frac{1-0.6}{1+0.6}} = 0.5\ \mathrm{y}^{-1}.$$

That is Fred would receive one signal every two years of his time. The first sixteen signals sent by Ethel during her onward journey to the star would be received by Fred in 32 years of his clock. During Ethel's return journey the frequency of signals sent by Ethel as measured by Fred would be

$$\nu' = \left(1\ \mathrm{y}^{-1}\right) \times \sqrt{\frac{1+0.6}{1-0.6}} = 2\ \mathrm{y}^{-1}.$$

Therefore, the other 16 signals sent by Ethel during her return trip would be received by Fred in a lapse of 8 years as shown in his clock. Therefore, Fred would conclude that Ethel has celebrated only 32 birthdays during lapse of 40 years of his life.

Therefore, there is no paradox as both conclude that after Ethel’s round trip that whereas Fred’s clock shows a lapse of 40 years, her clock shows a lapse of 32 years.