585.

Problem 42.24 (RHK)

A radioactive nucleus moves with a uniform velocity of $0.050c$ *in the laboratory frame. It decays by emitting a gamma ray. We have to find the direction of propagation of the gamma ray in the laboratory frame. We may assume that the gamma ray is emitted* (a) *parallel to the direction of motion of the nucleus, as seen in the frame in which the nucleus is at rest,* (b) *at* 45^0 *to this direction, and* (c) *at* 90^0 *to this direction.*

Solution:

Let S' be the inertial frame of reference in which the radioactive nucleus is at rest. It is given that with respect to the laboratory frame of reference S the frame S' is moving with a uniform velocity of $0.050c$. According to the special theory of relativity, if a gamma ray is emitted in the inertial frame S' at an angle θ' with respect to the direction of its relative velocity with respect to S, the angle of emission of gamma ray θ measured by S is related by the aberration formula

$$\tan\theta = \frac{\sin\theta'\sqrt{1-u^2/c^2}}{\cos\theta' + u/c}.$$

The velocity of S' with respect to S is 0.050c. We will, therefore, use the following equation for answering the three parts of the problem:

$$\tan\theta = \frac{\sin\theta'\sqrt{1-0.05^2}}{\cos\theta' + 0.05} = \frac{0.998\sin\theta'}{\cos\theta' + 0.05}.$$

(a)

For $\theta' = 0$, that is when the gamma ray is emitted in the forward direction, the laboratory frame will also observe that it is moving in the forward direction, $\theta = 0$.

(b)

For $\theta' = 45^0$, the laboratory frame will observe that the gamma ray is moving at angle given by the equation

$$\tan\theta = \frac{0.998 \times 0.707}{0.707 + 0.05} = \frac{0.705}{0.757} = 0.934$$

and

$$\theta = \tan^{-1}(0.934) = 43^0.$$

(c)

For $\theta' = 90^0$, the laboratory frame will observe that the gamma ray is moving at angle given by the equation

$$\tan\theta = \frac{0.998}{0.05},$$

and

$$\theta = \tan^{-1}\left(\frac{0.998}{0.05}\right) = 87.1^{0} \; .$$