584.

Problem 42.23 (RHK)

A source of light, at *rest in the* S' *frame, emits radiation uniformly in all directions*. (a) *We have to show that the fraction of light emitted into a cone of half-angle* θ' *is given by*

$$f = \tfrac{1}{2}\left(1 - \cos\theta'\right) \ .$$

We have to calculate f for $\theta' = 30^0$. (b) *When the source is viewed from S, the relative velocity of the two frames being* $0.80c$, we *have to find the value of* θ (*in frame S*) *using the relativistic aberration formula*

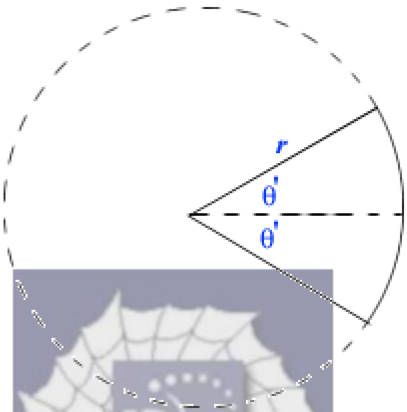
$$\tan\theta = \frac{\sin\theta'\sqrt{1 - u^2/c^2}}{\cos\theta' + u/c} \ .$$

We have to repeat the calculation for $u/c = 0.90$ *and* $u/c = 0.990$. *From our calculations we have to conclude why the aberration phenomenon is often referred to as the* "*headlight effect*".

Solution:

(a)

The area spanned by a cone of half-angle θ' on the surface of a sphere of radius *r* will be given by the integral



$$A = \int_0^{\theta'} (2\pi r \sin\theta) \times r d\theta$$

$$= 2\pi r^2 \left[-\cos\theta'\right]_0^{\theta'} = 2\pi r^2 (1 - \cos\theta) \ .$$

As the source of light is emitting radiation uniformly in all directions it will be distributed uniformly in area $4\pi r^2$ at a distance *r* from the source. Therefore, the fraction of light contained in the cone of half-angle θ' will be

$$f = \frac{A}{4\pi r^2} = \frac{1}{2}(1 - \cos\theta') \ .$$

For $\theta' = 30^0$ the f will be

$$f\left(30^0\right) = \frac{1}{2}(1 - 0.866) = 0.067.$$

(b)

Using the relativistic aberration formula, we calculate next the angle θ for $\theta' = 30^0$; $u/c = 0.80$, $u/c = 0.90$ and $u/c = 0.990$.

The relativistic aberration formula is

$$\tan\theta = \frac{\sin\theta'\sqrt{1 - u^2/c^2}}{\cos\theta' + u/c} .$$

For $\theta' = 30^0$, we have

$$\tan\theta = \frac{0.5 \times \sqrt{1 - (u/c)^2}}{0.866 + u/c}.$$

We thus find that for $u/c = 0.80$,

$\theta = 10.2^0$;

for $u/c = 0.90$,

$\theta = 7^0$;

and for $u/c = 0.990$,

$\theta = 2^0$.

The above calculations reveal that as the speed of the source increases the width of the light cone as observed by a stationary observer narrows down, as happens in a moving “head light”.