**583.**

**<u>Problem 42.27 (RHK)</u>**

*A source of radio waves, rest frequency* 188 MHz, *is moving at a speed of* 0.717*c transverse to the line of sight to a detector*. *We have to find the angle to the line of sight of a second source*, *rest frequency* 162 MHz, *moving also at* 0.717*c*, *for the frequencies of the two sources as received at the detector to be equal*.

**<u>Solution:</u>**

If a source of light moving with velocity $\vec{u}$ with respect to an inertial observer in a frame *S* emits light of frequency $\nu_0$ measured in the stationary frame of the source and its frequency $\nu$ is measured in the frame *S* when the line of site makes an angle $\theta$ with the velocity $\vec{u}$, the frequencies $\nu$ and $\nu_0$ are related by the relativistic Doppler effect equation

$$\nu = \frac{\nu_0 \sqrt{1-u^2/c^2}}{1-(u/c)\cos\theta} \ .$$

The frequency of the radio waves, rest frequency 188 MHz, emitted by a source moving with speed 0.717*c* at right angles to the line of site will be

$$\nu = 188 \times \sqrt{1 - 0.717^2} \text{ MHz} .$$

We have to find the angle to the line of sight of a second source, rest frequency 162 MHz, moving also at 0.717*c*, for the frequencies of the two sources as received at the detector to be equal. Let the angle between the line of site of the second source and its velocity be $\theta$. The Doppler shifted frequency as observed will be

$$\nu' = \frac{162 \times \sqrt{1 - 0.717^2}}{1 - 0.717\cos\theta} \text{ MHz} .$$

The condition that

$$\nu' = \nu,$$

gives the equation

$$\frac{162 \times \sqrt{1 - 0.717^2}}{1 - 0.717\cos\theta} = 188 \times \sqrt{1 - 0.717^2}.$$

Or

$$188 \times \left(1 - 0.717\cos\theta\right) = 162,$$

or

$$\cos\theta = \frac{26}{0.717 \times 188}.$$

$$\therefore \ \theta = \cos^{-1}\left(0.192\right) = 78.8^0 .$$