582.

Problem 42.25 (RHK)

We have to calculate the Doppler shift $\lambda - \lambda_0$, *if any, for the sodium* D_2 *line* (589.00 nm) *emitted from a source moving in a circle with constant speed* $0.122c$ *as measured by an observer fixed at the centre of the circle*.

Solution:

The relativistic Doppler frequency ν for light of frequency ν_0 emitted from a source moving with velocity $\vec{u}$ such that the angle between $\vec{u}$ and the direction of observation as seen by the observer is θ is given by the expression

$$\nu = \nu_0 \frac{\sqrt{1-u^2/c^2}}{1-(u/c)\cos\theta} .$$

As the source is moving in a circle and the frequency of the emitted light is measured by an observer stationary at the centre of the circle, the angle between the velocity of the source and the line of observation is $\pi/2$. This Doppler shift is of second order and is called the

transverse Doppler shift. As the source is rotating with constant speed of 0.122*c*, we have

$$\nu = \nu_0 \sqrt{1 - \frac{u^2}{c^2}},$$

or

$$\lambda = \frac{\lambda_0}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{589}{\sqrt{1 - 0.122^2}} \text{ nm} = 593.43 \text{ nm}.$$

Therefore,

$$\lambda - \lambda_0 = 4.43 \text{ nm}.$$