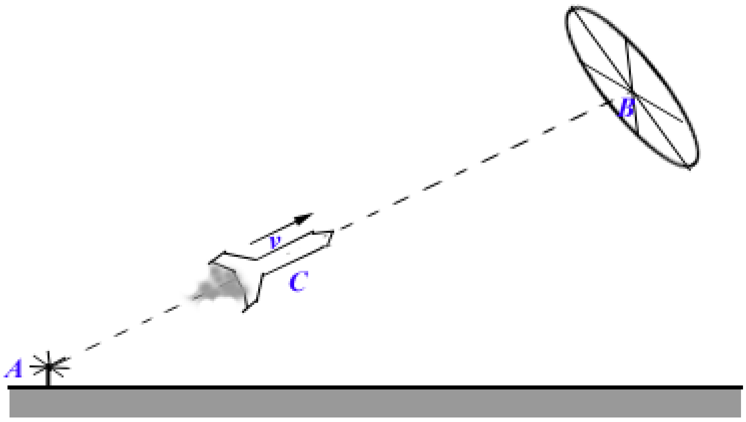
580.

Problem 42.19 (RHK)

A, on Earth, signals with a flash light every 6 min. *B is on a space station that is stationary with respect to Earth. C is on a rocket travelling from A to B with constant velocity of* $0.6c$ *relative to A; as shown in the figure.* (a) *We have to calculate the time intervals between the signals emitted by A as they are received by B and C.* (b) *If C flashes a light every time a flash is received from A, we have to find the time interval between the flashes emitted by C as received by B.*



Solution:

Space station *B* is stationary with respect to the observer *A* on Earth. An observer C is on a rocket that is travelling with speed 0.6*c* relative to *A*.

From *A* flash light signals are emitted every six minutes which travel to the space ship along the line of sight from *A* to *B*.

As *B* is stationary with respect to *A*, the time interval between the pulses received at *B* will also be six minutes. We calculate the time interval, *T*, as measured by *A* between the signals as they reach the observer on the rocket.

In the frame of reference of S let the co-ordinates be indicated by the pair (x,t), where x is the distance from *A* on the line of sight. Let the co-ordinate of the event when a signal is emitted by *A* be $(0,0)$ and that of this event when this signal reaches *C* be (x,t). As the signal travels with the speed of light *c*,

$x = ct.$

The coordinates of the event when the next flash signal is emitted at A after a lapse of $6 \text{ min} = 360 \text{ s}$ will be

$(0, 360 \text{ s})$. Let the observer *A* note that this signal in his frame of reference is received by *C* at time $(t+T)$ s. Therefore, the coordinates of the event when the second signal reaches *C* will be $(x+vT, t+T)$. As we are considering light signals

$$x + vT = c(t + T - 360 \text{ s}),$$

or

$$vT = c(T - 360 \text{ s}),$$

or

$$T = \frac{360c}{c-v} \text{ s}.$$

As $v = 0.6c$, we find

$$T = \frac{360}{0.4} \text{ s} = 900 \text{ s}.$$

This time interval as measured in the moving reference frame of *C* will be given by the Lorentz time dilatation relation. In C's clock the interval T will be measured to be

$$T_C = T\sqrt{1-(v/c)^2} = 900 \times \sqrt{1-0.6^2} \text{ s} = 720 \text{ s} = 12 \text{ min}.$$

(c)

If *C* flashes a light every time a light flash is received from *A*, the situation is equivalent to absence of *C*.

Therefore, the time interval between the light flashes as these arrive at *B*, which is stationary with respect to *A*, will also be 6 min.