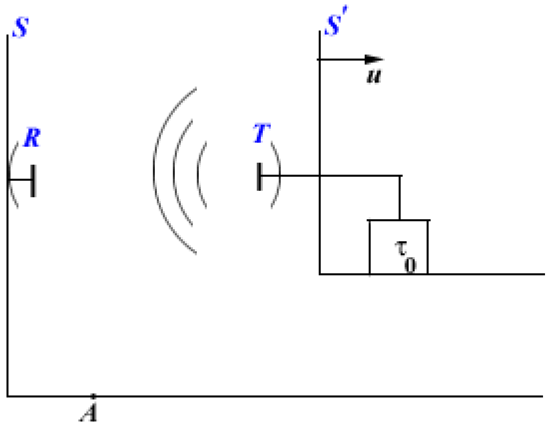
579.

Problem 42.20 (RHK)

A radar transmitter T is fixed to a reference frame S' *that is moving to the right with speed u relative to the reference frame S* (*see figure*). *A mechanical timer* (*essentially a clock*) *in frame* S', *having a period* τ_0 (*measured in* S') *causes transmitter T to emit radar pulses, which travel at the speed of light and are received by R, a receiver fixed in frame S.* (a) *We have to find the period* τ *of the timer relative to observer A, who is fixed in frame S.* (b) *We have to show that the receiver R would observe the time interval between pulses arriving from T, not as* τ *or as* τ_0, *but as*

$$\tau_R = \tau_0 \sqrt{\frac{c+u}{c-u}}.$$

(c) *We have to explain why the observer at R measures a different period for the transmitter than measured by observer A, who is in the same reference frame.*



Solution:

(a)

As shown in the figure reference frame S' is moving to the right with speed u relative to the reference frame S. The mechanical timer in S' is emitting radar pulses at time intervals τ_0. The time period of the timer as measured by an observer fixed in S, say A, will be given by the Lorentz dilatation relation

$$\tau = \frac{\tau_0}{\sqrt{1-(u/c)^2}}.$$

(b)

We next calculate the period of radar pulses measured by the receiver R of S.

We note that as the transmitter of radar waves is moving with speed u with respect to S, it would have moved by distance $u\tau$ to the right by the time transmitter in

S' emits successive radar pulse. Therefore, the time period of pulses measured by the receiver R of S will not be τ but

$\tau + u\tau/c$,

as the radar pulse travel with speed of light *c*. Therefore, the period of light pulses measured by the radar fixed in *S* will be

$$\tau_R = \tau\left(1+\frac{u}{c}\right) = \frac{\tau_0}{\sqrt{1-\frac{u^2}{c^2}}} \times \left(1+\frac{u}{c}\right)$$

$$= \tau_0 \frac{\left(1+\frac{u}{c}\right)^{1/2}}{\left(1-\frac{u}{c}\right)^{1/2}} = \tau_0\sqrt{\frac{c+u}{c-u}}.$$

(c)

The difference between τ_R and τ or τ_0 is that the frequency of pulses emitted by S', which is $\nu_0 = 1/\tau_0$, is Doppler shifted with respect to S. The frequency of radar pulses measured by S, which is $\nu = 1/\tau_R$, and frequency ν_0 of electromagnetic waves as measured by S' are related by the Doppler relation

$$\nu = \nu_0 \sqrt{\frac{c-u}{c+u}}.$$