578.

Problem 42.13 (RHK)

In the spectrum of quasar 3C9, *some of the familiar hydrogen lines appear but they are shifted so far toward the red that their wavelengths are observed to be three times as large as that observed in the light from the hydrogen atom at rest in the laboratory*. (a) *We have to show that the classical Doppler shift equation, which assumes that light behaves like sound, gives a velocity of recession greater than c*. (b) *Assuming that the relative motion of* 3C9 *and the Earth is entirely one of recession*; *we have to find the recession speed predicted by the relativistic Doppler equation*.

Solution:

(a)

The classical Doppler shift equation for source receding from the observer with speed u is

$$\nu = \nu_0 \frac{1}{1+u/c},$$

or

$$\lambda = \lambda_0 \left(1+u/c\right).$$

It is observed that in the spectrum of quasar 3C9 the wavelengths of the hydrogen lines are three times as large as observed in the laboratory. Assuming that light behaves like sound waves, and applying the classical Doppler shift equation, we find

$$1+u/c = \lambda/\lambda_0 = 3,$$

and

$$u = 2c.$$

The receding space of the quasar estimated using the classical Doppler equation is 2*c*, which is in violation of the relativity principle that the upper limit of speeds is the speed of light *c*.

We next apply the relativistic Doppler equation for estimating the receding speed of quasar 3C9.

(b)

For source and observer receding with relative speed *u*, the relativistic Doppler equation is

$$\nu = \nu_0 \sqrt{\frac{1-u/c}{1+u/c}},$$

or

$$\lambda = \lambda_0 \sqrt{\frac{1+u/c}{1-u/c}}.$$

As it is observed that for quasar 3C9,

$$\frac{\lambda}{\lambda_0} = 3,$$

applying relativistic Doppler equation, we get

$$\frac{1+u/c}{1-u/c} = 9,$$

or

$$10u/c = 8,$$

and

$$u = 0.80c.$$