576.

Problem 42.7 (RHK)

Consider a star located on a line perpendicular to the plane of the Earth's orbit about the Sun. The distance to the star is much greater than the diameter of the Earth's orbit. We have to show that, due to the finite speed of light, a telescope through which the star is seen must be tilted at an angle $\alpha = 20.5''$ *to the perpendicular, in the direction the Earth is moving. This phenomenon, called aberration, is noticeable and was first explained by James Bradley in 1729.*

Solution:

The orbital speed of the Earth in the frame of reference of the star is

$v = 29.8$ km s^{-1}.

Let us call the frame of reference in which star is at rest S' and the frame of reference in which Earth is moving in the x direction, which is parallel to the x' direction, S. The *relativistic equation for the aberration of light* is

$$\tan\theta = \frac{\sin\theta'\sqrt{1-\beta^2}}{\cos\theta' + \beta},$$

where θ and θ' are the directions of propagation of light as seen from the frames S and S', respectively.

As the light from the star reaching the Earth is being emitted in the $-y'$ direction

$$\theta' = \frac{3\pi}{2}.$$

Therefore,

$$\tan\theta = -\frac{\sqrt{1-\beta^2}}{\beta}.$$

As

$$\beta = \frac{v}{c} = \frac{29.8}{3\times10^5} = 9.93\times10^{-5},$$

$$\tan\theta \cong \theta = -9.93\times10^{-5}\ \text{rad} = -\left(\frac{9.93\times10^{-5}\times180\times3600}{\pi}\right)''$$

$$= -20.48''.$$

That is the telescope through which the star is to be seen needs to be tilted at an angle $20.5''$ from the $-y$ direction, that is the vertical, toward the direction of the Earth's orbital velocity.