573.

Problem 41.49 (RHK)

A particle in the solar system is under the combined influence of the Sun's gravitational attraction and the radiation force due to the sun's rays. We may assume that the particle is a sphere of density 1.00 g cm^{-3} *and that the entire incident light is absorbed.* (a) *We have to show that all particles with radius less than some critical radius* R_0 *will be blown out of the solar system.* (b) *We have to calculate* R_0. *We may note that* R_0 *does not depend on the distance from the particle to the Sun.*

Solution:

We will use the following astronomical data for answering this problem:

Mass of the Sun, $M_{sun} = 1.99 \times 10^{30}$ kg,

Rate of energy emission from the Sun,

$P = 3.90 \times 10^{26}$ W.

The mass of a particle of radius R_0 (m) made out of material of density 1.00 g cm^{-3} will be

$$m = \frac{4\pi}{3} R_0^3 \times 1.0 \times 10^3 \text{ kg.}$$

Let the distance to the Sun from the particle be r (m).

The force of gravity on the particle due to the Sun will be

$$F_{grav} = \frac{GM_{sun}m}{r^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times \left(\frac{4\pi}{3} R_0^3\right) \times 10^3}{r^2} \text{ N}$$

$$= \frac{5.56 \times 10^{23} \times R_0^3}{r^2} \text{ N.}$$

The intensity of the solar radiation at the particle will be

$$I = \frac{P}{4\pi r^2}.$$

As the entire solar radiation incident on the particle is absorbed, the magnitude of the force on the particle due to radiation will be

$$F_{rad} = \frac{I}{c} \times \left(\pi R_0^2\right) = \frac{3.96 \times 10^{26} \times \left(\pi R_0^2\right)}{4\pi r^2 \times 3 \times 10^8} \text{ N}$$

$$= \frac{0.33 \times 10^{18} \times R_0^2}{r^2} \text{ N.}$$

For this particle to get blown away from the solar system, the gravitational pull on it has to be less than the force due to absorption of solar radiation. We have the condition

$$F_{grav} < F_{rad},$$

or

$$\frac{5.56\times10^{23}\times R_0^3}{r^2}\ \text{N} < \frac{0.33\times10^{18}\times R_0^2}{r^2}\ \text{N},$$

or

$$R_0 < \frac{0.33\times10^{-5}}{5.56}\ \text{m} = 593\ \text{nm}.$$