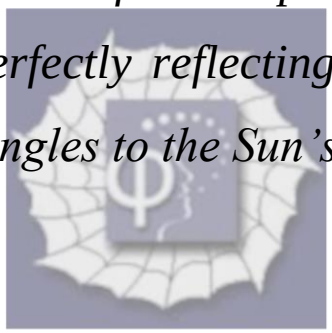
572.

Problem 41.47 (RHK)

It has been proposed that a spaceship might be propelled in the solar system by radiation pressure, using a large sail made of foil. We have to determine the size of the sail if the radiation force is to be equal in magnitude to the Sun's gravitational attraction. We may assume that the mass of the ship + sail is 1650 kg *and that the sail is perfectly reflecting, and that the sail is oriented at right angles to the Sun's rays*.



Solution:

We will use the following astronomical data for answering this problem:

Mass of the Sun, $M_{sun} = 1.99 \times 10^{30}$ kg,

Rate of energy emission from the Sun,

$P = 3.90 \times 10^{26}$ W.

Let the spaceship be at a distance R (m) from the Sun.

The mass of the spaceship plus that of the sails m is 1650 kg.

Therefore, the gravitational pull of the Sun on the space ship will be

$$F_{grav} = \frac{GM_{sun}m}{R^2} = \frac{6.67\times10^{-11}\times1.99\times10^{30}\times1650}{R^2}\ \text{N}$$

$$= \frac{2.19\times10^{23}}{R^2}\ \text{N.}$$

Let the area of the sails be $A\ \left(\text{m}^2\right)$. As the sails are perfectly reflecting the force experienced by the spaceship due to the radiation pressure will be

$$F_{rad} = \frac{2IA}{c},$$

where

I is the intensity of the solar radiation at the spaceship. It will be

$$I = \frac{P}{4\pi R^2}.$$

Therefore,

$$F_{rad} = \frac{2IA}{c} = \frac{2\times3.90\times10^{26}\times A}{3\times10^{8}\times4\pi\times R^2}\ \text{N} = \frac{2.069\times10^{17}A}{R^2}\ \text{N.}$$

Size of the sail A is to be fixed by requiring that the magnitude of the gravitational pull of the Sun on the spaceship and that of the radiation pressure be equal. That is

$$F_{rad} = F_{grav}.$$

This condition fixes the value of *A*. We have

$$\frac{2.069\times10^{17}\,A}{R^2}\ \text{N}=\frac{2.19\times10^{23}}{R^2}\ \text{N},$$

or

$$A=\frac{2.19}{2.069}\times10^{6}\ \text{m}^2=1.06\ \text{km}^2.$$