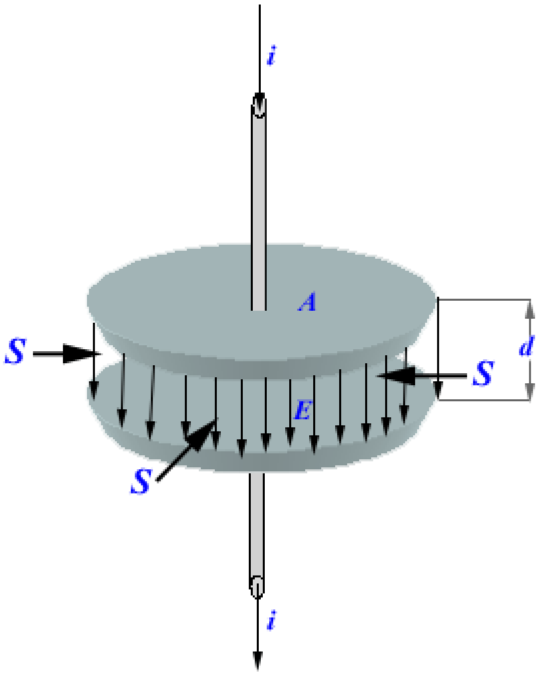
562.

<u>Problem 41.32 (RHK)</u>

In the figure a parallel-plate being charged is shown. (a) *We have to show that the Poynting vector* $\vec{S}$ *points everywhere radially into the cylindrical volume*; (b) *that the rate at which the energy flows into this volume, calculated by integrating over the cylindrical boundary of this volume, is equal to the rate at which the stored electrostatic energy increases; that is*

$$\int \vec{S}.d\vec{A} = Ad\frac{d}{dt}\left(\frac{1}{2}\varepsilon_0 E^2\right),$$

where Ad *is the volume of the capacitor and* $\frac{1}{2}\varepsilon_0 E^2$ *is the energy density for all points within the volume. This analysis shows that, according to the Poynting vector point of view, the energy stored in a capacitor does not enter it through the wire but through the space around the wire and the plates.*



Solution:

We will ignore the fringing of the electric field $\vec{E}$ at the boundary of the circular plates and assume that the electric field is confined to the gap between the plates as shown in the figure. That is it is uniform and perpendicular to the plates as shown in the figure.

The magnetic field at the circular boundary of the parallel-plate capacitor can be obtained by applying the Ampere's law modified for the displacement current arising due to the changing electric field during the charging process of the capacitor.

Let a be the radius of the circular plates. We note that the magnetic field at the boundary of the circular-plates will be circular. Applying the modified Ampere's law, we get

$$2\pi a B(a) = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\pi a^2 E(t)\right),$$

or

$$B(a) = \frac{\mu_0 \varepsilon_0 a}{2}\frac{d}{dt}\left(E(t)\right).$$

The direction of the magnetic field during the charging is circular and clockwise as seen from the top of the parallel-plate capacitor.

Hence, the Poynting vector $\vec{S} = (1/\mu_0)\vec{E}\times\vec{B}$, on the cylindrical boundary of the capacitor, will point radially inward. The magnitude of $\vec{S}$ will therefore be

$$\left|\vec{S}\right| = \frac{1}{\mu_0}EB = \frac{a}{2}\varepsilon_0 E\frac{d}{dt}(E) = \frac{a}{4}\frac{d}{dt}\left(\varepsilon_0 E^2\right).$$

We calculate the integral of the Poynting vector over the cylindrical bounding surface between the two plates. As the Poynting vector is normal to the surface the integral

$$\int \vec{S}.d\vec{A}$$

will be equal to the product of $\left|\vec{S}\right|$ and the area $A = 2\pi a d$. We thus have the result

$$\int \vec{S}.d\vec{A} = \left(2\pi ad\right)\times\frac{a}{4}\frac{d}{dt}\left(\varepsilon_0 E^2\right) = \left(\pi a^2 d\right)\frac{d}{dt}\left(\frac{\varepsilon_0}{2}E^2\right)$$

$$= Ad\,\frac{d}{dt}\left(\frac{\varepsilon_0}{2}E^2\right).$$

We recall that $\frac{1}{2}\varepsilon_0 E^2$ is the volume density of energy in the space between the plates of a parallel-plate capacitor. We may thus conclude that the energy stored between the plates of a parallel-plate capacitor does not enter it through the wires connected to the plate but through the space around the wires and the plates.