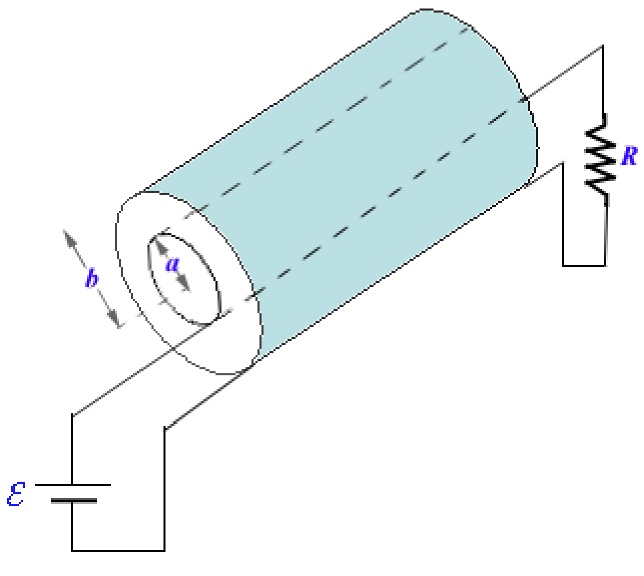
561.

<u>Problem 41.31 (RHK)</u>

A coaxial cable (*inner radius a, outer radius b*) *is used as a transmission line between a battery* $\mathcal{E}$ *and a resistance R, as shown in the figure.* (a) *We have to calculate E, B for* $a < r < b$; (b) *calculate the Poynting vector* $\vec{S}$ for $a < r < b$; (c) *by suitably integrating the Poynting vector show that the total power flowing across the annular cross section* $a < r < b$ *is* $\mathcal{E}^2/R$. *We have to answer whether this is reasonable.* (d) *We have to show that the direction of* $\vec{S}$ *is always from the battery to the resistor, no matter which way the battery is connected.*



Solution:

(a)

As the outer and the inner surfaces of the coaxial cable are connected to a battery of emf $\mathcal{E}$, the electric field inside the region between the two current carrying parts of the cable will be radial and of magnitude

$$E = \frac{\mathcal{E}}{b-a},$$

where b is the outer radius and a is the inner radius of the cable.

As the cable forms a circuit with a resistor of resistance R connected to the battery, there will be a current flow of magnitude

$$i = \frac{\mathcal{E}}{R}, \quad \text{for } a < r < b.$$

Because of the current flow in the inner part of the coaxial cable and the cylindrical symmetry of the cable there will be a circular magnetic field about the axis of the cable of magnitude

$$B(r) = \frac{\mu_0 i}{2\pi r}, \quad \text{for } a < r < b.$$

(b)

The magnetic field vector $\vec{B}(r)$ and the electric field vector $\vec{E}$ will be perpendicular to each other and it can be seen that $\vec{E}\times\vec{B}$ will be parallel to the axis of cable pointing in the direction of the flow of current from the battery to the resistor. Therefore, the magnitude of $\vec{E}\times\vec{B}$ at radial distance *r* from the axis of the cable will be

$$\left|\vec{E}\times\vec{B}\right| = EB = \frac{\mathcal{E}}{b-a}\times\frac{\mu_0 i}{2\pi r}, \quad \text{for } a<r<b.$$

(c)

Integrating the Poynting vector $\vec{S} = (1/\mu_0)\vec{E}\times\vec{B}$ over any annular cross section of the cable, we can obtain the power flowing inside the cable. We find

$$\int \vec{S}.d\vec{A} = \frac{\mathcal{E}i}{(b-a)}\int_a^b \frac{1}{2\pi r}\times 2\pi r dr = \mathcal{E}i.$$

As

$$i = \frac{\mathcal{E}}{R},$$

the power flowing inside the cable

$$\int \vec{S}.d\vec{A} = \frac{\mathcal{E}^2}{R}$$

is equal to the Joule dissipation of the energy by the resistor.