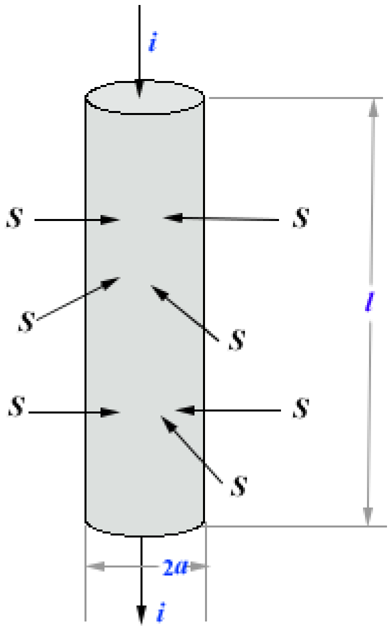
560.

Problem 41.30 (RHK)

In the figure a cylindrical resistor of length l, radius a, and resistivity ρ, *carrying a current i has been shown.* (a) *We have to show that the Poynting vector* $\vec{S}$ *at the surface of the resistor is everywhere directed normal to the surface, as shown.* (b) *We have to show that the rate at which the energy flows into the resistor through its cylindrical surface, calculated by integrating the Poynting vector over this surface, is equal to the rate at which the internal energy is produced; that is*

$$\int \vec{S}.d\vec{A} = i^2 R,$$

where $d\vec{A}$ *is an element of the area of the cylindrical surface. This suggests that, according to the Poynting vector point of view, the energy that appears in a resistor as internal energy does not enter it through the connecting wires but through the space around the wires and the resistor.*



<u>Solution:</u>

(a)

Let l be the length of the cylindrical resistor, a be its radius, and ρ be the resistivity of the conducting material. The resistance of the resistor is given by the expression

$$R = \frac{\rho l}{\pi a^2}.$$

Because of the cylindrical geometry the magnetic field $\vec{B}$ at the surface of the conductor will be circular and its direction will be clockwise as the current is flowing lengthwise from above. Applying the Ampere's law we note that the magnitude of the magnetic field at the surface of the cylindrical conductor will be

$$B = \frac{\mu_0 i}{2\pi a}.$$

The electric field $\vec{E}$ in the conductor will be in the direction of the current and its magnitude will be

$$E = \rho j$$
$$= \frac{\rho i}{\pi a^2}.$$

As $\vec{E}$ and $\vec{B}$ fields at the surface of the conductor are orthogonal, the Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

will point toward the axis of the cylindrical conductor in radial direction. The magnitude of $\vec{S}$ will therefore be

$$S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \times \frac{\rho i}{\pi a^2} \times \frac{\mu_0 i}{2\pi a} = \frac{\rho i^2}{2\pi^2 a^3}.$$

(b)

We will calculate

$$\int \vec{S}.d\vec{A}$$

over the surface of the cylinder of length l and radius a.

As $\vec{S}$ and $d\vec{A}$ are orthogonal, we have

$$\int \vec{S}.d\vec{A} = 2\pi alS = 2\pi al \times \frac{\rho i^2}{2\pi^2 a^3} = \frac{\rho l}{\pi a^2} \times i^2 = Ri^2.$$

This suggests that, according to the Poynting vector point of view, the energy that appears in a resistor as internal energy does not enter it through the connecting wires but through the space around the wires and the resistor.