559.

Problem 41.29 (RHK)

We consider the possibility of standing electromagnetic waves

$$E = E_m(\sin\omega t)(\sin kx),$$
$$B = B_m(\cos\omega t)(\cos kx).$$

We have to show (a) *that these satisfy*

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t},$$
$$\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0\frac{\partial E}{\partial t},$$

if E_m *is suitably related to* B_m *and* ω *is suitably related to k*. (b) *We have to find the* (*instantaneous*) *Poynting vector*. (c) *We have to show the time-average power flow across any area is zero*. (d) *We have to describe the flow of energy in this situation*.

Solution:

We describe the standing electromagnetic waves by the functions

$$E = E_m(\sin \omega t)(\sin kx),$$
$$B = B_m(\cos \omega t)(\cos kx).$$

From these functions we obtain the following partial derivatives:

$$\frac{\partial E}{\partial x} = E_m k \sin \omega t \cos kx,$$
$$\frac{\partial B}{\partial t} = -B_m \omega \sin \omega t \cos kx,$$
$$\frac{\partial B}{\partial x} = -B_m k \cos \omega t \sin kx,$$

and

$$\frac{\partial E}{\partial t} = E_m \omega \cos \omega t \sin kx.$$

We require that the partial derivates of the *E* and *B* fields satisfy the relations

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t},$$
$$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}.$$

We note that we get the equations

$$E_m k \sin \omega t \cos kx = B_m \omega \sin \omega t \cos kx,$$

and

$$B_m k \cos \omega t \sin kx = \mu_0 \varepsilon_0 E_m \omega \cos \omega t \sin kx.$$

These equations will be satisfied provided

$$E_m k = B_m \omega,$$

and

$$B_m k = \mu_0 \varepsilon_0 E_m \omega,$$

or

$$\frac{E_m}{B_m} = \frac{\omega}{k}, \quad \left(\frac{\omega}{k}\right)^2 = \frac{1}{\mu_0 \varepsilon_0} = c^2,$$

where c is the speed of electromagnetic waves in vacuum.

Therefore,

$$\frac{E_m}{B_m} = c.$$

(b)

Assuming that the $\vec{E}$ and $\vec{B}$ fields are perpendicular, the magnitude of the Poynting vector S will be given by the expression

$$S = \frac{1}{\mu_0} E_m B_m \sin \omega t \cos \omega t \sin kx \cos kx$$

$$= \frac{1}{4\mu_0} E_m B_m \sin 2\omega t \cos 2kx.$$

(c)

Time average of S will be

$$\bar{S} = \frac{1}{4\mu_0}\langle \sin 2\omega t \rangle \sin 2kx.$$

As

$$\langle \sin 2\omega t \rangle = 0,$$

$$\bar{S} = 0.$$

(d)

There is no flow of energy along the length and we have standing waves.