558.

Problem 41.28 (RHK)

A copper wire ($diameter = 2.48$ mm; $resistance$ 1.00 Ω *per* 300 m) *carries a current of* 25.0 A. *We have to calculate* (a) *the electric field*, (b) *the magnetic field*, *and* (c) *the Poynting vector magnitude for a point on the surface of the wire*.

Solution:

(a)

Electric field *E*, the resistivity ρ, and the current density *j* are related as

$$\rho = \frac{E}{j},$$

and the resistance *R* of a conducting wire of length l, cross-sectional area *A*, and resistivity ρ is given by the relation

$$R = \frac{\rho l}{A}.$$

Therefore,

$$E = \frac{jRA}{l} = \frac{iR}{l},$$

where i the current flowing through a conductor of cross-sectional area A is jA. It is given that the resistance per unit length of the copper wire

$$\frac{R}{l} = \frac{1}{300}\ \Omega\ \text{m}^{-1}.$$

Therefore, the electric field at the surface of the wire will be

$$E = 25.0 \times \frac{1}{300}\ \text{V m}^{-1} = 8.33 \times 10^{-2}\ \text{V m}^{-1}.$$

(b)

We will use the Ampere's law for finding the magnetic field at the surface of the wire. If the radius of the wire is r, the magnetic field B and the current i are related as

$$2\pi r B(r) = \mu_0 i.$$

Therefore,

$$B(r) = \frac{\mu_0 i}{2\pi r} = \frac{2 \times 10^{-7} \times 25}{1.24 \times 10^{-3}}\ \text{T} = 4.03 \times 10^{-3}\ \text{T}.$$

(c)

As the electric field $E(r)$ and the magnetic field $B(r)$ are orthogonal at the surface of the current carrying wire, the magnitude of the Poynting vector will be given by

$$S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \times \frac{iR}{l} \times \frac{\mu_0 i}{2\pi r} = \frac{i^2 R}{2\pi r l}$$

$$= \frac{25^2}{300 \times 2\pi \times 1.24 \times 10^{-3}} \text{ W m}^{-2}$$

$$= 267.4 \text{ W m}^{-2}.$$