557.

Problem 41.27 (RHK)

The average intensity of sunlight, falling at normal incidence just outside the Earth's atmosphere, varies during the year due to the changing Earth-Sun distance. We have to show that the fractional yearly variation is given by $\Delta I/I = 4e$ *approximately, where e the eccentricity of the Earth's elliptical orbit around the Sun.*

Solution:

Let *a* be the semi-major axis of the Earth's elliptic orbit and *e* be the eccentricity of the orbit.

The aphelion (the maximum distance of the Earth from the Sun) is given by

$$r_{ap} = a(1+e)$$

and the perihelion (the minimum distance of the Earth from the Sun) is given by

$$r_{pe} = a(1-e).$$

The maximum variation in intensity of the sunlight will therefore be

$$\Delta I = I\left(\text{perihelion}\right) - I\left(\text{aphelion}\right)$$

$$= \frac{\alpha}{a^2\left(1-e\right)^2} - \frac{\alpha}{a^2\left(1+e\right)^2}$$

$$= \frac{\alpha\left(\left(1+e\right)^2 - \left(1-e\right)^2\right)}{a^2\left(1-e^2\right)^2} ; \; \frac{4e\alpha}{a^2},$$

where α is a constant related to the Sun's radiating power.

As average intensity of the sunlight at the Earth is given by

$$I = \frac{\alpha}{a^2},$$

we note that

$$\frac{\Delta I}{I} = 4e.$$