554.

Problem 41.20 (RHK)

Sunlight strikes the Earth, just outside its atmosphere, with an intensity of 1.38 kW m^{-2}. *Assuming sunlight to be a plane wave, we have to calculate* (a) E_m *and* (b) B_m *for it.*

Solution:

For the plane electromagnetic wave, the magnitude of the Poynting vector, *S*, is

$$S = \frac{1}{\mu_0} EB,$$

which can also be written, using $E = cB$,

$$S = \frac{1}{\mu_0 c} E^2 \quad \text{or} \quad S = \frac{c}{\mu_0} B^2,$$

where *S*, *E*, and *B* are instantaneous values at the observation point. The time average $\overline{S}$ is also known as the intensity *I* of the wave. We have

$$I = \overline{S} = \frac{1}{\mu_0 c} \overline{E}^2 = \frac{1}{\mu_0 c} E_m^2 \left\langle \sin^2 \left(kx - \omega t \right) \right\rangle.$$

The time average of $\sin^2$ over any whole number of cycles is $1/2$. Therefore,

$$I = \bar{S} = \frac{1}{\mu_0 c}\bar{E}^2 = \frac{1}{2\mu_0 c}E_m^2.$$

As the intensity of the sunlight outside the Earth's atmosphere is given to be

$I = 1.40 \times 10^3$ W m^{-2},

The electric field E_m associated with the plane wave will be

$$E_m = \left(2\mu_0 c I\right)^{1/2} = \left(2\times 4\pi \times 10^{-7} \times 3\times 10^8 \times 1.40\times 10^3\right)^{1/2} \text{ V m}^{-1}$$

$$= 1.02 \text{ kV m}^{-1}.$$

And the magnetic field associated with the wave will be given by

$$B_m = \frac{E_m}{c} = \frac{1.02\times 10^3}{3\times 10^8} \text{ T} = 3.4 \ \mu\text{T}.$$