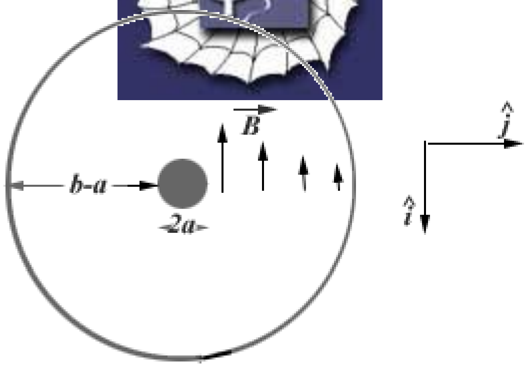
521.

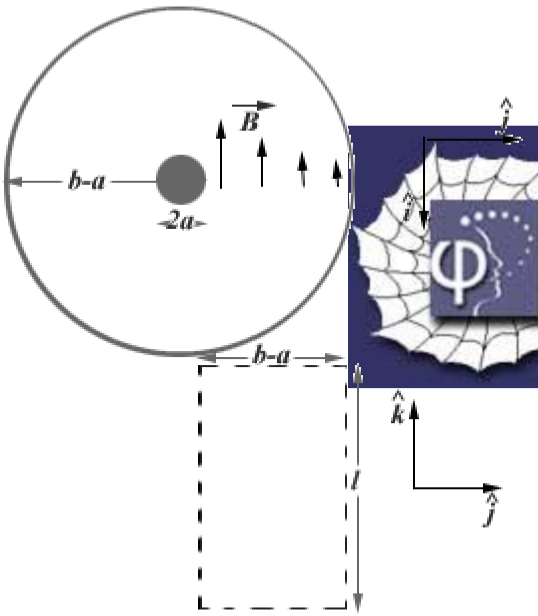
<u>Problem 38.15 (RHK)</u>

A long coaxial cable consists of two concentric cylindrical conductors with radii a and b, where b ? a. *Its central conductor carries a steady current* i, *and the outer conductor provides the return path. We have to find the inductance of a length l of this cable by calculating the flux through a rectangular surface, of length l and width* $b-a$.



Solution

A long coaxial cable consists of two concentric cylindrical conductors with radii a and b, where b ? a. Its central conductor carries a steady current i, and the outer conductor provides the return path. We have to find the inductance of a length *l* of this cable. The magnetic field inside the coaxial cable will depend on the distance from the central axis. By using Ampere's law, we note that the magnetic field at a distance r from the axis of central cable when a current i is flowing through it will be



$$B(r)=\frac{\mu_0 i}{2\pi r},\quad \text{for } a<r<b.$$

We consider a Gaussian surface, a rectangle of length *l* and width $b-a$ in the $\hat{j}\hat{k}-plane$. The direction of the magnetic field on this surface will be normal to it. Therefore, the flux enclosed by the rectangular Gaussian surface of width $b-a$ and length l will be given by the integral

$$\Phi = \int_a^b l dr B(r) = \int_a^b l dr \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i l}{2\pi} \ln\left(\frac{b}{a}\right).$$

As the inductance is flux per unit current, the inductance of a length l of the coaxial cable will therefore be

$$L = \frac{\Phi}{i} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right).$$