

550.

Problem 38.22 (HR)

We have to calculate expressions for the following four quantities, considering both $r < R$ and $r > R$:

- (a) $B(r)$ for a current i in a long wire of radius R ;
- (b) $E(r)$ for a long uniform cylinder of charge of radius R ;
- (c) $B(r)$ for a parallel-plate capacitor, with circular plates of radius R , in which E is changing at a constant rate; and
- (d) $E(r)$ for a cylindrical region of radius R in which a uniform magnetic field is changing at constant rate.

Solution:

(a)

For finding $B(r)$ for a current i in a long wire of radius

R we will use Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i.$$

Case $r > R$:

$$2\pi rB(r) = \mu_0 i,$$

or

$$B(r) = \frac{\mu_0 i}{2\pi r}.$$

Case $r < R$:

$$2\pi rB(r) = \mu_0 i \times \left(\frac{\pi r^2}{\pi R^2} \right),$$

or

$$B(r) = \frac{\mu_0 i}{2\pi} \times \left(\frac{r}{R^2} \right).$$

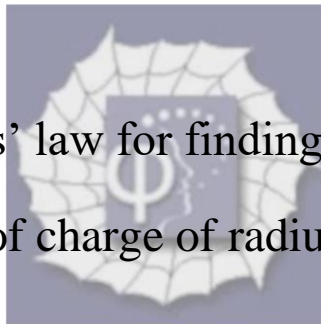
(b)

We will use Gauss' law for finding $E(r)$ for a long uniform cylinder of charge of radius R .

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}.$$

Case $r > R$:

Let the charge per unit length on the cylindrical wire of radius R be λ . We consider a cylindrical Gaussian surface of length l and radius r . We have



$$2\pi r l E(r) = \frac{\lambda l}{\epsilon_0},$$

or

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}, \text{ for } r > R .$$

Case $r < R$:

$$2\pi r l E(r) = \left(\frac{\lambda}{\pi R^2} \right) \times \frac{(\pi r^2 l)}{\epsilon_0},$$

or

$$E(r) = \frac{\lambda}{2\pi\epsilon_0} \times \left(\frac{r}{R^2} \right) \text{ for } r < R.$$

(c)

We will use Ampere's law (version extended by Maxwell) for finding $B(r)$ for a parallel-plate capacitor, with circular plates of radius R , in which E is changing at a constant rate.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \left(\epsilon_0 \frac{d\Phi_E}{dt} \right).$$

Case $r > R$:

$$2\pi r B(r) = \mu_0 \epsilon_0 \times (\pi R^2) \times \frac{dE}{dt},$$

or

$$B(r) = \frac{\mu_0 \epsilon_0}{2} \times \frac{R^2}{r} \times \frac{dE}{dt} \text{ for } r > R .$$

Case $r < R$:

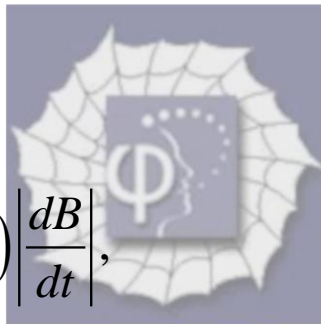
$$2\pi r B(r) = \mu_0 \varepsilon_0 \times (\pi r^2) \times \frac{dE}{dt},$$

or

$$B(r) = \frac{\mu_0 \varepsilon_0 r}{2} \times \frac{dE}{dt} \text{ for } r < R .$$

(d) We will find $E(r)$ for a cylindrical region of radius R in which a uniform magnetic field is changing at constant rate using Faraday's law

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}.$$



Case $r > R$:

$$2\pi r \left| \mathbf{E}(r) \right| = \left(\pi R^2 \right) \left| \frac{dB}{dt} \right|,$$

or

$$\left| \mathbf{E}(r) \right| = \frac{R^2}{2r} \left| \frac{dB}{dt} \right| \text{ for } r > R.$$

Case $r < R$:

$$2\pi r \left| \mathbf{E}(r) \right| = \left(\pi r^2 \right) \left| \frac{dB}{dt} \right|,$$

or

$$\left| \mathbf{E}(r) \right| = \frac{r}{2} \left| \frac{dB}{dt} \right| \text{ for } r < R .$$