550.

<u>Problem 38.22 (HR)</u>

We have to calculate expressions for the following four quantities, considering both $r < R$ and $r > R$:

(a) $B(r)$ *for a current* i *in a long wire of radius* R;

(b) $E(r)$ *for a long uniform cylinder of charge of radius* R;

(c) $B(r)$ *for a parallel-plate capacitor, with circular plates of radius* R, *in which* E *is changing at a constant rate; and*

(d) $E(r)$ *for a cylindrical region of radius* R *in which a uniform magnetic field is changing at constant rate.*

<u>Solution:</u>

(a)

For finding $B(r)$ for a current i in a long wire of radius R we will use Ampere's law

$$\oint \vec{B}.d\vec{s} = \mu_0 i.$$

Case $r > R$:

$$2\pi r B(r) = \mu_0 i,$$

or

$$B(r) = \frac{\mu_0 i}{2\pi r} \ .$$

Case $r < R$:

$$2\pi r B(r) = \mu_0 i \times \left(\frac{\pi r^2}{\pi R^2} \right),$$

or

$$B(r) = \frac{\mu_0 i}{2\pi} \times \left(\frac{r}{R^2} \right) \ .$$

(b)

We will use Gauss' law for finding $E(r)$ for a long uniform cylinder of charge of radius R.

$$\oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0} \ .$$

Case $r > R$:

Let the charge per unit length on the cylindrical wire of radius R be λ. We consider a cylindrical Gaussian surface of length l and radius r. We have

$$2\pi r l E(r) = \frac{\lambda l}{\varepsilon_0},$$

or

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}, \text{ for } r > R\ .$$

Case $r < R$:

$$2\pi r l E(r) = \left(\frac{\lambda}{\pi R^2}\right) \times \frac{\left(\pi r^2 l\right)}{\varepsilon_0},$$

or

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0} \times \left(\frac{r}{R^2}\right) \text{ for } r < R.$$

(c)

We will use Ampere's law (version extended by Maxwell) for finding $B(r)$ for a parallel-plate capacitor, with circular plates of radius R, in which E is changing at a constant rate.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left(\varepsilon_0 \frac{d\Phi_E}{dt}\right).$$

Case $r > R$:

$$2\pi r B(r) = \mu_0 \varepsilon_0 \times \left(\pi R^2\right) \times \frac{dE}{dt},$$

or

$$B(r) = \frac{\mu_0 \varepsilon_0}{2} \times \frac{R^2}{r} \times \frac{dE}{dt} \text{ for } r > R\ .$$

Case $r < R$:

$$2\pi r B(r) = \mu_0 \varepsilon_0 \times \left(\pi r^2\right) \times \frac{dE}{dt},$$

or

$$B(r) = \frac{\mu_0 \varepsilon_0 r}{2} \times \frac{dE}{dt} \text{ for } r < R \ .$$

(d) We will find $E(r)$ for a cylindrical region of radius R in which a uniform magnetic field is changing at constant rate using Faraday's law

$$\oint \vec{E} . d\vec{s} = -\frac{d\Phi_B}{dt}.$$

Case $r > R$:

$$2\pi r \left|\vec{E}(r)\right| = \left(\pi R^2\right) \left|\frac{dB}{dt}\right|,$$

or

$$\left|\vec{E}(r)\right| = \frac{R^2}{2r} \left|\frac{dB}{dt}\right| \text{ for } r > R.$$

Case $r < R$:

$$2\pi r \left|\vec{E}(r)\right| = \left(\pi r^2\right) \left|\frac{dB}{dt}\right|,$$

or

$$\left|\vec{E}(r)\right| = \frac{r}{2} \left|\frac{dB}{dt}\right| \text{ for } r < R \ .$$