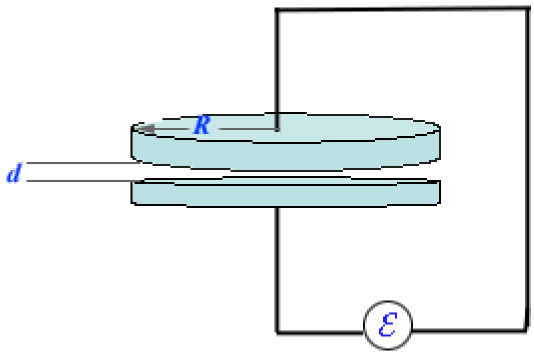
**549.**

**Problem 40.14 (RHK)**

*A capacitor consisting of two circular plates with radius* $R = 18.2\ \mathrm{cm}$ *is connected to a source of emf* $\mathrm{E} = \mathrm{E}_m \sin \omega t$, *where* $\mathrm{E}_m = 225\ \mathrm{V}$ *and* $\omega = 128\ \mathrm{rad\ s^{-1}}$. *The maximum value of the displacement current is* $i_d = 7.63\ \mu\mathrm{A}$. *Neglecting fringing of the electric field at the edges of the plates, we have to find the maximum value of the current i; the maximum value of* $d\Phi_E/dt$, *where* $\Phi_E$ *is the electric flux through the region between the plates*; the *separation between the plates d; and the maximum value of the magnitude of* $\vec{B}$ *between the plates at a distance* $r = 11.0\ \mathrm{cm}$ *from the centre.*



**Solution:**

From the data of the problem we note that the emf applied across the plates of the capacitor can be described by the function

$$\mathrm{E} = 225 \sin 128t \text{ V}.$$

Let the separation between the plates be $d$ m. Variation of the electric field between the plates will be given by the function

$$E = \frac{225}{d} \sin 128t \text{ V m}^{-1}.$$

The displacement current $i_d$ is given in terms of the change of electric flux per unit time by the relation

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$

The maximum value of $d\Phi_E / dt$ will therefore be given by

$$\left(\frac{d\Phi_E}{dt}\right)_{\max} = \frac{(i_d)_{\max}}{\varepsilon_0}.$$

It is given that

$$(i_d)_{\max} = 7.63 \ \mu\text{A}.$$

Therefore,

$$\left(\frac{d\Phi_E}{dt}\right)_{\max} = \frac{7.63\times10^{-6}}{8.854\times10^{-12}} \text{ V m s}^{-1}$$

$$= 0.8617\times10^{6} \text{ V m s}^{-1}$$

$$= 861.7 \text{ kV m s}^{-1}.$$

The maximum value of the charging current will be equal to the maximum value of the displacement current. Therefore, the maximum value of the charging current will be $7.63\ \mu\text{A}$.

We will next calculate the separation between the plates. We know that

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0\left(\pi R^2\right)\frac{dE}{dt} = \frac{\varepsilon_0\left(\pi R^2\right)}{d}\frac{dV}{dt}$$

$$= \frac{\varepsilon_0\left(\pi R^2\right)}{d}\times 225\times 128\cos 128t.$$

Therefore,

$$\left(i_d\right)_{\max} = \frac{\varepsilon_0\left(\pi R^2\right)}{d}\times 225\times 128 \text{ A},$$

and

$$d = \frac{\varepsilon_0\left(\pi R^2\right)\times 225\times 128}{\left(i_d\right)_{\max}} \text{ m}$$

$$= \frac{8.854\times10^{-12}\times\pi\times\left(18.2\times10^{-2}\right)^2\times 225\times 128}{7.63\times10^{-6}} \text{ m}$$

$$= 3.47 \text{ mm}.$$

We will find next the maximum value of $\vec{B}$ between the plates at a distance $r = 11.0\ \text{cm}$ from the centre. The Ampere's law states that

$$\oint \vec{B}.d\vec{s} = \mu_0 i_d .$$

Considering an Amperian circular loop of radius $r = 11.0\ \text{cm}$, we have

$$\left(B(r)\right)\times\left(2\pi r\right) = \mu_0 i_d ,$$

and

$$\left(B(r)\right)_{\max} = \frac{\mu_0 \left(i_d\right)_{\max}}{2\pi r}\times\left(\frac{\pi r^2}{\pi R^2}\right)$$

$$= \frac{\left(4\pi\times10^{-7}\right)\times7.63\times10^{-6}}{2\pi\times11.0\times10^{-2}}\times\left(\frac{11}{18.2}\right)^2\ \text{T}$$

$$= 5.06\times10^{-12}\ \text{T}.$$