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Problem 40.13 (RHK)

Suppose that a circular-plate capacitor has a radius R of 32.1 mm *and a plate separation of* 4.80 mm. *A sinusoidal potential difference with a maximum value of* 162 V *and a frequency of* 60.0 Hz *is applied between the plates. We have to find the maximum value of the induced magnetic field at $r = R$.*

Solution:

From the data of the problem we note that the potential can be described by the function

$$V = 162\sin\left(2\pi \times 60t\right) \text{ V},$$

and

$$\frac{dV}{dt} = 162 \times \left(2\pi \times 60\right)\cos\left(120\pi t\right) \text{ V s}^{-1}$$

$$= 61.07 \times 10^{3} \cos\left(120\pi t\right) \text{ V s}^{-1} .$$

In a parallel-plate capacitor with plate separation *d*, the electric field E and the potential difference V are related as

$$E = \frac{V}{d},$$

we have

$$\frac{dE}{dt} = \frac{61.07\times10^{3}}{4.80\times10^{-3}}\cos\left(120\pi t\right)\text{V s}^{-1}$$
$$=12.72\times10^{6}\ \cos\left(120\pi t\right)\text{V s}^{-1}.$$

We note that

$$\left(\frac{dE}{dt}\right)_{\max} =12.72\times10^{6}\ \text{V m}^{-1}\ \text{s}^{-1}.$$

Ampere’s law is

$$\oint \vec{B}.d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}.$$

Applying it to an Amperian loop of radius $r = R$, we have

$$2\pi RB\left(R\right)= \mu_0\varepsilon_0 \times\left(\pi R^{2}\right)\times\frac{dE}{dt},$$

or

$$B\left(R\right)= \mu_0\varepsilon_0 \times\frac{R}{2}\times\frac{dE}{dt}.$$

The maximum value of the magnetic field between the plates at $R = 32.1\times10^{-3}$ m will therefore be

$$
\begin{aligned}
\left(B(R)\right)_{\max} &= \mu_0 \varepsilon_0 \times \frac{R}{2} \times \left(\frac{dE}{dt}\right)_{\max} \\
&= \frac{32.1\times 10^{-3}}{2\times\left(3\times 10^{8}\right)^{2}} \times 12.72\times 10^{6}\ \text{T} \\
&= 2.27\ \text{pT}.
\end{aligned}
$$