Problem 40.12 (RHK)

In 1929 M. R. Van Cauwenberghe succeeded in measuring directly, for the first time, the displacement current i_d *between the plates of a parallel plate capacitor to which an alternating potential difference was applied. He used circular plates whose effective radius was* 40.0 cm *and whose capacitance was* 100 pF. *The applied potential difference had a maximum value* V_m *of* 174 kV *at a frequency of* 50.0 Hz. (a) *We have to calculate the maximum displacement current that was present between the plates.* (b) *We have to answer why the applied potential difference was chosen to be so high.* (*The delicacy of these measurements is such that they were only performed in a direct manner more than 60 years after Maxwell enunciated the concept of displacement current.*)

Solution:

The displacement current is defined by the relation

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt},$$

where the flux due to the electric field between the plates of the capacitor of radius R is

$$\Phi_E = \left(\pi R^2\right) E \ .$$

Let d be the separation between the plates and let V be the potential difference between the plates. As the electric field between the plates is uniform, we have

$$E = \frac{V}{d}.$$

The capacitance of a parallel plate (circular plates) capacitor is

$$C = \frac{\varepsilon_0}{d}\left(\pi R^2\right).$$

Therefore,

$$\Phi_E = \frac{CV}{\varepsilon_0} \ ,$$

and

$$i_d = C\frac{dV}{dt}.$$

In the experiment the potential difference varied simple harmonically with frequency $\nu = 50$ Hz and maximum value V_m of 174 kV. We therefore have

$V = 174\times10^3 \sin\left(2\pi\times50t\right)$ V,

and

$$\frac{dV}{dt} = 174\times10^3\times2\pi\times50\cos\left(2\pi\times50t\right) \text{ V s}^{-1}.$$

Therefore, the maximum value of dV/dt in the experiment was

$$\left(\frac{dV}{dt}\right)_{\max} = 5.46\times10^7 \text{ V s}^{-1}.$$

In the experiment capacitance of the capacitor used was

$C = 100\times10^{-12}$ F $= 10^{-10}$ F.

Therefore, the maximum displacement current present in the experiment was

$$\left(i_d\right)_{\max} = 10^{-10}\times5.46\times10^7 \text{ A} = 5.46 \text{ mA}.$$

(b)

The displacement current is equal to product of the capacitance and the rate of change with time of the potential between the plates, and as the capacitance of the condenser was of the order of pico Farad, for the displacement current to be of the order of mA the potential difference had to be of the order of kV.