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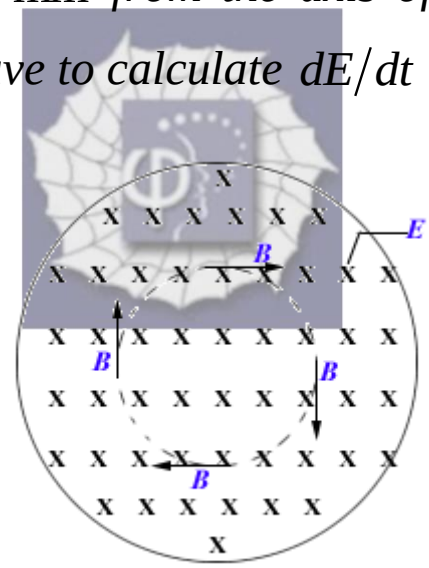
Problem 40.11 (RHK)

A parallel-plate capacitor with circular plates 21.6 cm in diameter is being charged. The displacement current density throughout the region is uniform, into the paper in the diagram, and has a value of $1.87\ \text{mA cm}^{-2}$. *We have to calculate* (a) *the magnetic field B at a distance* $r = 53.0\ \text{mm}$ *from the axis of symmetry of the region.* (b) *We have to calculate* dE/dt *in this region.*



Solution:

(a)

Ampere's law as extended by Maxwell is

$$\oint \vec{B}.d\vec{s} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt},$$

$$= \mu_0 i + \mu_0 i_{d,}$$

Where

$\left(\text{displacement current}\right)\ i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}.$

There will not be conduction current between the plates of a capacitor; therefore, in this region magnetic field will be due to the displacement current only. We consider an Amperian loop of radius $r = 53.0\ \text{mm}.$ Applying the Maxwell modified Ampere's law, we get

$$B \times \left(2\pi \times 53.0 \times 10^{-3}\right) = \mu_0 \left(\frac{1.87 \times 10^{-3}}{10^{-4}} \times \pi \times \left(53.0 \times 10^{-3}\right)^2 \right),$$

or

$$B = \mu_0 \times \frac{1.87 \times 10}{2} \times 53.0 \times 10^{-3}\ \text{T},$$

$$= \left(4\pi \times 10^{-7}\right) \times 1.87 \times 5 \times 53.0 \times 10^{-3}\ \text{T} = 623\ \text{nT}.$$

(b)

We calculate next dE/dt in this region.

$$\varepsilon_0 \frac{dE}{dt} = \frac{1.87 \times 10^{-3}}{10^{-4}}\ \text{A m}^{-2},$$

or

$$\frac{dE}{dt} = \frac{18.7}{\varepsilon_0}\ \text{V m}^{-1}\ \text{s}^{-1} = \frac{18.7}{8.854 \times 10^{-12}}\ \text{V m}^{-1}\ \text{s}^{-1}$$

$$= 2.11 \times 10^{12}\ \text{V m}^{-1}\ \text{s}^{-1}.$$