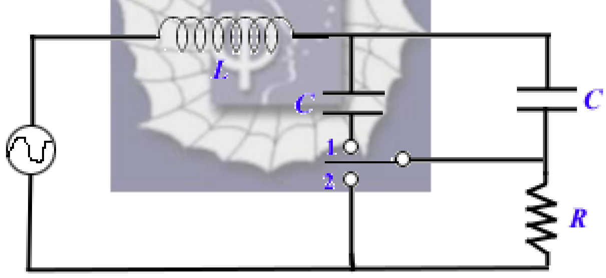
542.

Problem 39.37 (RHK)

The AC generator shown in the figure supplies 170 V (*max*) *at* 60 Hz. *With the switch open as in the diagram, the resulting current leads the generator emf by* 20^0. *With the switch in position* 1 *the current lags the generator emf by* 10^0. *When the switch is in position* 2 *the maximum current is* 2.82 A. *We have to find the values of R, L, and C.*



Solution:

Let the AC emf and the current in the circuit be described by the functions

$$\mathrm{E} = \mathrm{E}_m \sin \omega t,$$

and

$$i = i_m \sin(\omega t - \phi).$$

The phase angle ϕ is given in terms of the values of the circuit components *R*, *L*, and *C*, and the angular frequency ω by the relation

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}.$$

With the switch open as in the diagram, the resulting current leads the generator emf by 20^0. Therefore,

$$\tan\left(-20^0\right) = \frac{\omega L - 1/\omega C}{R},$$

or

$$-0.364R = \omega L - 1/\omega C. \quad \text{(A)}$$

It is given that when the switch in position 1 the current lags the generator emf by 10^0. Therefore,

$$\tan\left(10^0\right) = \frac{\omega L - 1/2\omega C}{R},$$

or

$$0.176R = \omega L - 1/2\omega C. \quad \text{(B)}$$

The additional data is that when the switch is in position 2 the maximum current is 2.82 A. This implies that

$$\frac{170}{\left|\left(L\omega - 1/C\omega\right)\right|} = 2.82,$$

or

$$\left|\left(L\omega - 1/C\omega\right)\right| = \frac{170}{2.82}\ \Omega = 60.28\ \Omega. \quad \text{(C)}$$

From equations (C) and (A), we find the value of the resistance R. We find

$$R = \frac{60.28}{0.364}\ \Omega = 165.6\ \Omega.$$

Substituting the value of R in (B), we get

$$\omega L - 1/2\omega C = 0.176 \times 165.6\ \Omega = 29.15\ \Omega.$$

Substituting the value of R in (A), we get

$$1/\omega C - L\omega = 60.28\ \Omega,$$

$$\therefore \frac{1}{2C\omega} = (60.28 + 29.15)\ \Omega = 89.43\ \Omega,$$

and

$$C = \frac{1}{2\times(2\pi)\times 60 \times 89.43}\ \mathrm{F} = 14.8\ \mu\mathrm{F}.$$

And,

$$L\omega = 118.58\ \Omega,$$

$$\therefore L = \frac{118.58}{2\times(2\pi)\times 60}\ \mathrm{H} = 315\ \mathrm{mH}.$$