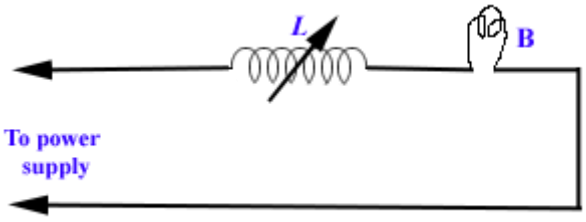
541.

Problem 39.36 (RHK)

A typical "light dimmer" used to dim the stage lights in a theatre consists of a variable inductor L connected in series with the light bulb B *as shown in the figure. The power supply is* 120 V (*rms*) *at* 60.0 Hz; *the light bulb is marked* "120 V, 1000 W." (a) *We have to find the maximum inductance L required if the power in the light bulb is to be varied by a factor of five. We may assume that the resistance of the light bulb is independent of its temperature.* (b) *We have to answer whether a variable resistor instead of an inductor could have been used. If so, what maximum resistance is required? Why isn't this done?*



Solution:

(a)

We will find the resistance of the light bulb from its ratings “120 V, 1000 W”.

In a purely resistive load

$$\bar{P} = \frac{\mathrm{E}_{rms}^2}{R}.$$

$$\therefore\ R = \frac{(120)^2}{1000}\ \Omega = 14.4\ \Omega.$$

When the inductor is connected in series with the light bulb, the power dissipation is given by the equation

$$\bar{P} = \mathrm{E}_{rms} i_{rms} \cos\phi,$$

where

$$\cos\phi = \frac{R}{\sqrt{R^2 + (L\omega)^2}},$$

and

$$i_{rms} = \frac{\mathrm{E}_{rms}}{Z} = \frac{\mathrm{E}_{rms}}{\sqrt{R^2 + (L\omega)^2}}.$$

In order that power in the light bulb could be varied by a factor of five, the maximum value of the required inductance will be given by the equation

$$\frac{\mathrm{E}_{rms}^2 R}{R^2 + L_{\max}^2 \omega^2} = \frac{1000}{5}.$$

Solving this equation, we find

$L_{\max} = 76.4$ mH.

(b)

If the power in the light bulb is to be dimmed by using a variable resistor instead of an inductor, the equation that will fix the maximum value of the additional resistance r that will be required is

$$\left(\frac{\mathrm{E}_{rms}}{R+r}\right)^2 \times R = 200,$$

or

$$(14.4+r) = \frac{120\times\sqrt{14.4}}{\sqrt{200}}\ \Omega = 32.2\ \Omega,$$

or

$r = 17.8\ \Omega.$

Yes, a variable resistor instead of a variable inductor can be used as a “light-dimmer”, but then more power will be drawn from the mains, which would get dissipated as heat.