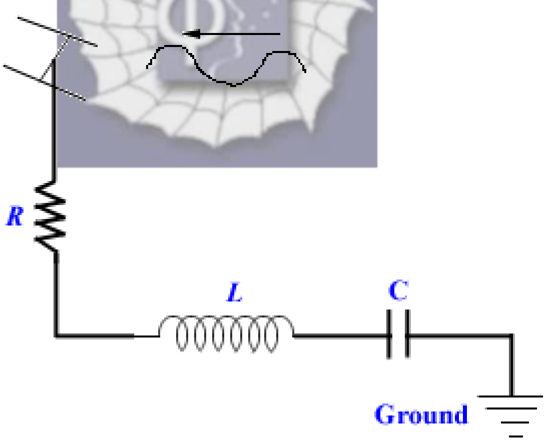
540.

Problem 39.30 (RHK)

Consider the FM antenna circuit shown in the figure, with $L = 8.22\ \mu\text{H}$, $C = 0.270\ \text{pF}$, *and* $R = 74.7\ \Omega$. *The radio signal induces an alternating emf in the antenna with* $\mathcal{E}_{rms} = 9.13\ \mu\text{V}$. *We have to find* (a) *the frequency of the incoming waves for which the antenna is in "tune"*; (b) *the rms current in the antenna, and* (c) *the rms potential difference across the capacitor*.



Solution:

The inductance of the antenna

$L = 8.22\ \mu\text{H}$,

and the capacitance of the antenna

$C = 0.270\ \text{pF}$.

Therefore, the frequency of the incoming waves for which the antenna is in “tune” will be

$$\nu = \frac{1}{2\pi} \times \frac{1}{\sqrt{LC}} = \frac{1}{2\pi} \times \frac{1}{\sqrt{8.22\times10^{-6}\times0.270\times10^{-12}}} \text{ Hz}$$

$$= 106.8 \text{ MHz.}$$

The radio signal induces an alternating emf in the antenna with $\mathrm{E}_{rms} = 9.13\ \mu\mathrm{V}$. As the antenna is tuned to the frequency of the FM signal, the impedance will be resistive and as the resistance of the antenna $R = 74.7\ \Omega$, the rms current in the antenna will be

$$i_{rms} = \frac{\mathrm{E}_{rms}}{R} = \frac{9.13\times10^{-6}}{74.7} \text{ A} = 0.122\ \mu\text{A.}$$

The capacitive reactance of the antenna will be

$$X_C = \frac{1}{C\omega} = \sqrt{\frac{L}{C}} = \sqrt{\frac{8.22\times10^{-6}}{0.270\times10^{-12}}} = 5.52\times10^{3}\ \Omega.$$

The rms potential difference across the capacitor will be

$$V_C = i_{rms}X_C = 0.122\times10^{-6}\times5.52\times10^{3} \text{ V} = 0.67 \text{ mV.}$$