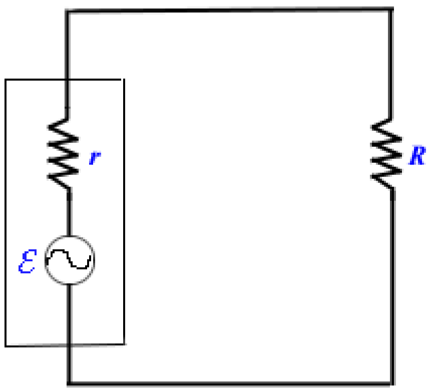
539.

<u>Problem 39.29 (RHK)</u>

We have to show that in the circuit shown in the figure the power dissipated in the resistance R is maximum when $R = r$, in which r is the internal resistance of the AC generator.



<u>Solution:</u>

Let the emf of the *AC*-generator be

$$\mathrm{E} = \mathrm{E}_m \sin \omega t\,.$$

As the total resistance in the resistance in the circuit is $R + r$, the current i in the circuit will be

$$i = \left(\frac{\mathrm{E}_m}{R + r}\right) \sin \omega t.$$

The average power dissipated in the circuit will be

$$\bar{P} = i_{rms}^2 R = \frac{1}{2} \times \left(\frac{\mathrm{E}_m}{R+r} \right)^2 R.$$

The function $\bar{P}(R)$ will have an extremum at R for which

$$\frac{d\bar{P}}{dR} = 0.$$

This condition gives

$$\frac{1}{(R+r)^2} - \frac{2R}{(R+r)^3} = 0,$$

or

$$R + r - 2R = 0,$$

or

$$R = r.$$

By calculating $\left.\frac{d^2\bar{P}(R)}{dR^2}\right|_{R=r}$, we will show that it is <0, and therefore $\bar{P}(R=r)$ is a maximum.

$$\left.\frac{d^2\bar{P}(R)}{dR^2}\right|_{R=r} = \frac{1}{2}\mathcal{E}_m^2 \left\{ -\frac{2}{(R+r)^3} - \frac{2}{(R+r)^3} + \frac{6R}{(R+r)^4} \right\}_{R=r}$$
$$= -\frac{\mathcal{E}_m^2}{16r^3} < 0.$$

The power dissipation is maximum when the external load is equal to the load of the generator.