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Problem 39.16 (RHK)

A resistor-inductor-capacitor combination R_1, L_1, C_1, *has a resonant frequency that is just the same as that of a different combination* R_2, L_2, C_2. *The two are next connected in combination in series. We have to show that this new circuit also has the same resonant frequency as the separate individual circuits.*

Solution:

The resonant frequencies of the two LCR circuits with circuit elements R_1, L_1, C_1 and R_2, L_2, C_2 are same.

Therefore,

$$\frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}} = \omega \ .$$

When the two LCR circuits are combined in series the inductance of the combination will be

$$L = L_1 + L_2 \ ,$$

and the capacitance of the combination will be

$$C = \frac{C_1 C_2}{C_1 + C_2} \ .$$

Therefore, the resonant frequency of the series combination of the two *LCR* circuits will be

$$\omega' = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(L_1 + L_2\right)\left(C_1 C_2 / C_1 + C_2\right)}}$$

$$= \frac{\sqrt{C_1 + C_2}}{\sqrt{(L_1 C_1)C_2 + (L_2 C_2)C_1}}$$

$$= \frac{\sqrt{C_1 + C_2}}{\sqrt{\frac{C_2}{\omega^2} + \frac{C_1}{\omega^2}}} = \omega \ .$$

Therefore,

The resonant frequency of the series combination of the two *LCR* circuits each of resonant frequency ω is also ω.