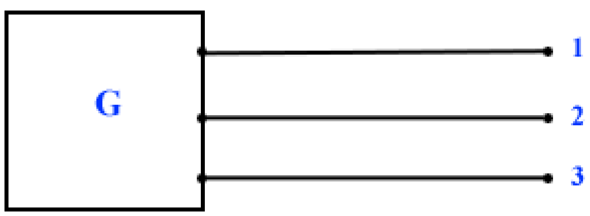
532.

Problem 39.8 (RHK)

A three-phase generator G produces electrical power that is transmitted by three wires as shown in the figure. The potentials (*relative to a common reference level*) *of these wires are* $V_1 = V_m \sin \omega t$, $V_2 = V_m \sin\left(\omega t - 120^0\right)$, *and* $V_3 = V_m \sin\left(\omega t - 240^0\right)$. *Some industrial equipment* (*for example, motors*) *has three terminals and is designed to be connected directly to these three wires. To use a more conventional two-terminal device* (*for example, a light bulb*), *one connects it to any tow of the three wires. We have to show that the potential difference between any two of the wires* (a) *oscillates sinusoidally with angular frequency* ω *and* (b) *has amplitude* $V_m\sqrt{3}$.



Solution:

The potentials (relative to a common reference level) of wires 1,2, and 3 are

$V_1 = V_m \sin \omega t$,

$V_2 = V_m \sin\left(\omega t - 120^0\right)$,

and $V_3 = V_m \sin\left(\omega t - 240^0\right)$.

The potential difference between wires 1 and 2 will therefore be

$$V_{12} = V_m \sin \omega t - V_m \sin\left(\omega t - 2\pi/3\right)$$
$$= V_m \left(\sin \omega t - \sin \omega t \cos\left(2\pi/3\right) + \cos \omega t \sin\left(2\pi/3\right)\right)$$
$$= V_m \left(\frac{3}{2}\sin \omega t + \frac{\sqrt{3}}{2}\cos \omega t\right) = \sqrt{3}V_m \left(\frac{\sqrt{3}}{2}\sin \omega t + \frac{1}{2}\cos \omega t\right)$$
$$= \sqrt{3}V_m \sin\left(\omega t + \pi/6\right).$$

Similarly, we can show that

$$V_{31} = V_m \sin\left(\omega t - 4\pi/3\right) - V_m \sin\left(\omega t\right)$$
$$= -\sqrt{3}V_m \cos\left(\omega t - 2\pi/3\right),$$

and

$$V_{23} = V_m \sin\left(\omega t - 2\pi/3\right) - V_m \sin\left(\omega t - 4\pi/3\right)$$
$$= -\sqrt{3}V_m \cos \omega t.$$

We note that V_{12}, V_{31}, and V_{23} oscillate sinusoidally with angular frequency ω and (b) have amplitude $V_m\sqrt{3}$.