530.

Problem 38.67 (RHK)

We have to find the resistance R that should be connected to an inductor $L = 220$ mH *and capacitor* $C = 12\ \mu$F *in series in order that the maximum charge on the capacitor decays to* 99% *of its initial value in* 50 *cycles.*

Solution:

In a LCR circuit the variation of charge as a function of time is given by the function

$$q = q_m e^{-Rt/2L} \cos\left(\omega' t + \phi\right),$$

in which

$$\omega' = \sqrt{\omega^2 - \left(R/2L\right)^2}\,.$$

The inductance in the circuit is

$$L = 220 \text{ mH},$$

and the capacitance in the circuit is

$$C = 12\ \mu\text{F}.$$

Therefore, the period of *LC*-oscillations will be

$$T = 2\pi\sqrt{LC} = 2\pi\sqrt{220\times10^{-3}\times12\times10^{-6}}\ \text{s} = 0.0102\ \text{s}.$$

We want to fix the resistance in the *LCR*-circuit in order that the maximum charge on the capacitor decays to 99% of its initial value in 50 cycles, that is in

$t = 50 \times 0.0102 \text{ s} = 0.51 \text{ s}.$

We have the condition

$$\frac{99}{100} = e^{-R \times 0.51 / 2 \times 220 \times 10^{-3}} = e^{-1.159R},$$

or

$$\ln(.99) = -1.159R,$$

$$\therefore \; R = \frac{1.005 \times 10^{-2}}{1.159} \; \Omega = 8.67 \text{ m}\Omega.$$