Problem 38.64 (RHK)

In the circuit shown in the figure the 900- μ F capacitor is initially charged to 100 V and the 100- μ F is uncharged. We have to describe in detail how one might charge the 100- μ F capacitor to 300 V by manipulating switches S_1 and S_2 .



Solution:

It is given that initially the 900- μ F capacitor is initially charged to 100 V and the 100- μ F is uncharged.

Therefore, an amount of energy

$$U = \frac{1}{2}CV^{2} = \frac{1}{2} \times 900 \times 10^{-6} \times (100)^{2} \text{ J} = 4.5 \text{ J}$$

is stored in the 900- μ F capacitor. The switch S_2 is closed and the switch S_1 is kept open for a time interval that will be determined next. The circuit is then a *LC*-oscillator with frequency

$$\omega_1 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 900 \times 10^{-6}}} \text{ rad s}^{-1} = 10.5 \text{ rad s}^{-1};$$

period

$$T_1 = \frac{2\pi}{10.05}$$
 s = 0.596 s.

After a lapse of time equal to $T_1/4 = 0.149$ s the energy which was initially stored in the 900- μ F capacitor will be transferred to the 10 H inductor and will be stored in it as magnetic energy. At this stage the switch S_2 is opened and the switch S_1 is closed. We now have a *LC*oscillator with frequency

$$\omega_2 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 100 \times 10^{-6}}} \text{ rad s}^{-1} = 31.6 \text{ rad s}^{-1};$$

period

$$T_2 = \frac{2\pi}{31.6}$$
 s = 0.198 s.

The energy in the 100- μ F when its potential is 300 V will be

$$U' = \frac{1}{2}CV^{2} = \frac{1}{2} \times 100 \times 10^{-6} \times (300)^{2} \text{ J} = 4.5 \text{ J}.$$

In time $T_2/4 = 0.05$ s the 4.5 J energy stored in the inductor will be transferred as electrical energy on to the $100-\mu$ F capacitor and the potential difference across it then will be 300 V.

