528.

Problem 38.57 (RHK)

In an oscillating LC circuit, in terms of the charge on the capacitor, we have to find (a) *the value of the charge present when the energy in the electric field is one-half that in the magnetic field*. (c) *We have to find the fraction of a period that must elapse following the time when the capacitor was fully charged for this condition to arise*.

Solution:

In an oscillating *LC* circuit variation of charge with time is sinusoidal and can be represented by the function

$$q = q_m \sin(\omega t + \phi),$$

and

$$i = \frac{dq}{dt} = q_m \omega \cos(\omega t + \phi).$$

Energy density in the electric field is

$$U_E = \frac{q^2}{2C},$$

and the energy in the magnetic field is

$$U_B = \frac{1}{2}Li^2.$$

Let the time be t when the energy in the electric field is one-half that in the magnetic field. We have the condition

$$\frac{q^2(t)}{2C} = \frac{1}{2}\left(\frac{1}{2}Li^2(t)\right),$$

or

$$\frac{q_m^{\ 2}}{2C}\sin^2(\omega t + \phi) = \frac{L}{4}\frac{q_m^{\ 2}}{LC}\cos^2(\omega t + \phi)$$

$$= \frac{q_m^{\ 2}}{4C}\left(1 - \sin^2(\omega t + \phi)\right).$$

This relation will be satisfied when

$$\sin^2(\omega t + \phi) = \frac{1}{3}.$$

The charge on the capacitor at this instant will be

$$q = \frac{q_m}{\sqrt{3}}.$$

Let t_0 be the instant when the capacitor was fully charged. At that instant,

$$(\omega t_0 + \phi) = \frac{\pi}{2}.$$

Let at $t = t_0 + \alpha T$,

$$\sin\left(\omega\left(t_0+\alpha T\right)+\phi\right)=\frac{1}{\sqrt{3}},$$

or

$$\cos\left(2\pi\alpha\right)=\frac{1}{\sqrt{3}},$$

and,

$$2\pi\alpha=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)=9.55\times10^{-1}\text{ rad.}$$

$$\therefore\ \alpha=0.152.$$

The fraction of a period that must elapse following the time when the capacitor was fully charged for this condition to arise will be $t/T=0.152$.