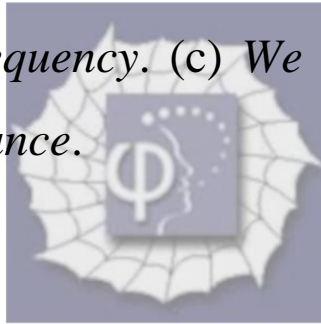


527.

Problem 38.53 (RHK)

An oscillating LC circuit is designed to operate at a peak current of 31 mA. The inductance of 42 mH is fixed and the frequency is varied by changing C. (a) If the capacitor has a maximum peak voltage of 50 V, we have to find whether the circuit can be safely operated at a frequency of 1.0 MHz. (b) We have to find the maximum safe operating frequency. (c) We have to determine the minimum capacitance.



Solution:

(a)

In a *LC* circuit the variation of charge with time on the capacitor is given by the function

$$q = q_m \cos(\omega t + \phi),$$

where

$$\omega = \frac{1}{\sqrt{LC}}.$$

The variation of the current in the circuit with time is given by the function

$$i = \frac{dq}{dt} = -\omega q_m \sin(\omega t + \phi).$$

Therefore, the values of the maximum charge and the maximum current are related as

$$i_m = \omega q_m.$$

The circuit has been designed to operate at a peak current of 31 mA. Therefore, the maximum safe charge in the circuit at a frequency of 1.0 MHz will be

$$q_m = \frac{i_m}{\omega} = \frac{31 \times 10^{-3}}{2\pi \times 10^6} \text{ C} = 4.93 \times 10^{-9} \text{ C}.$$

The inductance of 42 mH is fixed and the frequency is varied by changing C . The capacitance required for operating this system at frequency of 1 MHz can be found from the relation

$$\omega = \frac{1}{\sqrt{LC}},$$

or

$$C = \frac{1}{4\pi^2 \nu^2 L} = \frac{1}{4\pi^2 \times (10^6)^2 \times 42 \times 10^{-3}} \text{ F} = 6.03 \times 10^{-13} \text{ F}.$$

The maximum voltage across the capacitor at this frequency will be

$$V_m = \frac{q_m}{C} = \frac{4.93 \times 10^{-9}}{6.03 \times 10^{-13}} \text{ V} = 8.17 \times 10^3 \text{ V}.$$

As the peak voltage rating of the capacitor is 50 V, the circuit cannot be safely operated at 1.0 MHz.

(b)

We will next find the maximum safe operating frequency ω_m . We will use the following relations in finding answer to this part of the problem.

$$q_m = \frac{i_m}{\omega_m}; C = \frac{1}{L\omega_m^2}; \text{ and } q_m = 50C.$$

We therefore have the relation

$$\frac{i_m}{\omega_m} \times L\omega_m^2 = 50 \text{ V,}$$

or

$$\omega_m = \frac{50 \text{ V}}{Li_m} = \frac{50}{31 \times 10^{-3} \times 42 \times 10^{-3}} \text{ rad s}^{-1} = 3.84 \times 10^4 \text{ rad s}^{-1}.$$

And

$$\nu_m = \frac{\omega_m}{2\pi} = 6.11 \times 10^3 \text{ Hz.}$$

(c)

The minimum capacitance will therefore be

$$C_{\min} = \frac{1}{4\pi^2 L \nu_m^2} = \frac{1}{4\pi^2 \times (6.11 \times 10^3)^2 \times 42 \times 10^{-3}} \text{ F}$$

$$= 16.1 \text{ nF.}$$

