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Problem 38.53 (RHK)

An oscillating LC circuit is designed to operate at a peak current of 31 mA. *The inductance of* 42 mH *is fixed and the frequency is varied by changing C*. (a) *If the capacitor has a maximum peak voltage of* 50 V, *we have to find whether the circuit can be safely operated at a frequency of* 1.0 MHz. (b) *We have to find the maximum safe operating frequency*. (c) *We have to determine the minimum capacitance*.

Solution:

(a)

In a *LC* circuit the variation of charge with time on the capacitor is given by the function

$$q = q_m \cos(\omega t + \phi),$$

where

$$\omega = \frac{1}{\sqrt{LC}}.$$

The variation of the current in the circuit with time is given by the function

$$i = \frac{dq}{dt} = -\omega q_m \sin(\omega t + \phi).$$

Therefore, the values of the maximum charge and the maximum current are related as

$$i_m = \omega q_m.$$

The circuit has been designed to operate at a peak current of 31 mA. Therefore, the maximum safe charge in the circuit at a frequency of 1.0 MHz will be

$$q_m = \frac{i_m}{\omega} = \frac{31 \times 10^{-3}}{2\pi \times 10^6} \text{ C} = 4.93 \times 10^{-9} \text{ C}.$$

The inductance of 42 mH is fixed and the frequency is varied by changing C. The capacitance required for operating this system at frequency of 1 MHz can be found from the relation

$$\omega = \frac{1}{\sqrt{LC}},$$

or

$$C = \frac{1}{4\pi^2 \nu^2 L} = \frac{1}{4\pi^2 \times (10^6)^2 \times 42 \times 10^{-3}} \text{ F} = 6.03 \times 10^{-13} \text{ F}.$$

The maximum voltage across the capacitor at this frequency will be

$$V_m = \frac{q_m}{C} = \frac{4.93 \times 10^{-9}}{6.03 \times 10^{-13}} \text{ V} = 8.17 \times 10^3 \text{ V}.$$

As the peak voltage rating of the capacitor is 50 V, the circuit cannot be safely operated at 1.0 MHz.

(b)

We will next find the maximum safe operating frequency ω_m. We will use the following relations in finding answer to this part of the problem.

$$q_m = \frac{i_m}{\omega_m};\ C = \frac{1}{L\omega_m{}^2};\ \text{and } q_m = 50C.$$

We therefore have the relation

$$\frac{i_m}{\omega_m} \times L\omega_m{}^2 = 50 \text{ V},$$

or

$$\omega_m = \frac{50 \text{ V}}{Li_m} = \frac{50}{31\times10^{-3}\times42\times10^{-3}} \text{ rad s}^{-1} = 3.84\times10^4 \text{rad s}^{-1}.$$

And

$$\nu_m = \frac{\omega_m}{2\pi} = 6.11\times10^3 \text{ Hz}.$$

(c)

The minimum capacitance will therefore be

$$C_{\min} = \frac{1}{4\pi^2 L\nu_m{}^2} = \frac{1}{4\pi^2\times\left(6.11\times10^3\right)^2\times42\times10^{-3}} \text{ F}$$

$$= 16.1 \text{ nF}.$$