525.

Problem 38.43 (RHK)

A long wire carries a current i uniformly distributed over a cross section of the wire. (a) *We have to show that the magnetic energy of a length l stored within the wire equals* $\mu_0 i^2 l/16\pi$. (b) *We have to show that the inductance for a length l of wire associated with the flux inside the wire is* $\mu_0 l/8\pi$.

Solution:

Let the radius of the wire be *R*. It is given that it is carrying a current i. Therefore, the current density in the wire is

$$j = \frac{i}{\pi R^2}.$$

We use the Ampere's law for finding the magnetic field within the wire at a distance *r* from its axis. The magnetic field will be cylindrical. Therefore,

$$2\pi r B(r) = \mu_0 j \left(\pi r^2\right)$$

$$= \mu_0 \frac{i}{\pi R^2} \times \left(\pi r^2\right).$$

$$\therefore \; B(r) = \frac{\mu_0 i r}{2\pi R^2}.$$

Magnetic energy volume density is given by

$$U = \frac{B^2}{2\mu_0}.$$

Therefore, the energy density of the magnetic field at a distance r from the axis of the wire carrying current i will be

$$U(r) = \frac{B^2(r)}{2\mu_0} = \frac{\mu_0 i^2 r^2}{8\pi^2 R^4}.$$

The energy stored as magnetic field within a length l of the wire will therefore be given by the following integral:

$$\text{U} = \int_0^R 2\pi r l dr \times \frac{\mu_0 i^2 r^2}{8\pi^2 R^4}$$

$$= \frac{\mu_0 i^2 l}{4\pi R^4} \int_0^R r^3 dr = \frac{\mu_0 i^2 l}{16\pi}.$$

The inductance of a length l of the wire can now be found from the relationship

$$\text{U} = \frac{1}{2} L i^2.$$

Therefore,

$$L = \frac{\mu_0 l}{8\pi}.$$