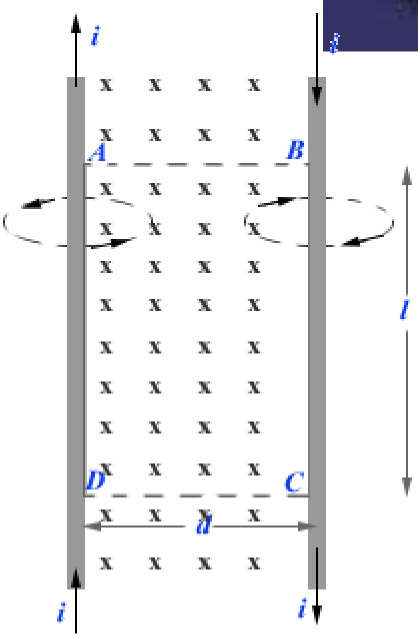
520.

Problem 38.13 (RHK)

Two long parallel wires, each of radius a, whose centres are a distance d apart carry equal currents in opposite directions. Show that, neglecting the flux within the wires themselves, the inductance of the length l of such a pair of wires is given by

$$L = \frac{\mu_0 l}{\pi} \ln\left(\frac{d-a}{a}\right).$$

Solution:



Two long parallel wires, each of radius a, whose centres are a distance d apart carry equal currents in opposite directions. Let us calculate the flux of the magnetic field through the rectangular area *ABCD* $\left(AD = BC = l; AB = DC = d\right)$.

We will find the magnetic field due to the two long current carrying wires. As the current is flowing in opposite directions, the magnetic field inside the rectangle *ABCD* will be perpendicular and into the plane of the figure (as shown in the figure).

The magnetic field at a distance r from a long wire carrying current i is circular and its magnitude is

$$B = \frac{\mu_0 i}{2\pi r}.$$

Therefore, the magnetic field due to both wires at a point a distance ξ from the left-end wire will be

$$B(\xi) = \frac{\mu_0 i}{2\pi\xi} + \frac{\mu_0 i}{2\pi(d-\xi)}.$$

The flux through the rectangular area *ABCD* will therefore be given by the integral

$$\begin{aligned}\Phi = \int_a^{d-a} l d\xi B(\xi) &= l\int_a^{d-a} d\xi \left(\frac{\mu_0 i}{2\pi}\right)\left(\frac{1}{\xi} + \frac{1}{d-\xi}\right) \\ &= \left(\frac{\mu_0 i l}{2\pi}\right) \times \left[\ln\left(\frac{d-a}{a}\right) + \ln\left(\frac{d-a}{a}\right)\right] \\ &= \frac{\mu_0 i l}{\pi}\ln\left(\frac{d-a}{a}\right).\end{aligned}$$

Inductance is defined as the flux per unit current. Therefore, the inductance of the length l of the pairs of wires will be

$$L = \frac{\mu_0 l}{\pi} \ln\left(\frac{d-a}{a}\right).$$