

516.

Problem 37.24 (RHK)

Consider a solid containing N atoms per unit volume, each atom having a magnetic dipole moment $\vec{\mu}$. Suppose the direction of $\vec{\mu}$ can be only parallel or antiparallel to an externally applied magnetic field \vec{B} (this will be the case if $\vec{\mu}$ is due to the spin of a single electron). According to statistical mechanics, it can be shown that the probability of an atom being in a state with energy U is proportional to $e^{-U/kT}$ where T is the temperature and k is the Boltzmann's constant. Thus since $U = -\vec{\mu} \cdot \vec{B}$, the fraction of atom whose dipole moment is parallel to \vec{B} is proportional to $e^{\mu B/kT}$ and the fraction of atoms whose dipole moment is antiparallel to \vec{B} is proportional to $e^{-\mu B/kT}$. (a) We have to show that the magnetization of this solid is $M = N\mu \tanh(\mu B/kT)$. (b) We have to show that (a) reduces to $M = N\mu^2 B/kT$ for $\mu B = kT$. (c) We have to show that (a) reduces to $M = N\mu$ for $\mu B \gg kT$.

Solution:

Consider a solid containing N atoms per unit volume, each atom having a magnetic dipole moment $\vec{\mu}$. We suppose the direction of $\vec{\mu}$ can be only parallel or antiparallel to an externally applied magnetic field \vec{B} (this will be the case if $\vec{\mu}$ is due to the spin of a single electron). According to statistical mechanics, it can be shown that the probability of an atom being in a state with energy U is proportional to $e^{-U/kT}$ where T is the temperature and k is the Boltzmann's constant.

As

$$U = -\vec{\mu} \cdot \vec{B},$$



the fraction of the atoms whose dipole moment is parallel

$$\text{to } \vec{B}, \frac{N_{\uparrow}}{N},$$

$$\frac{N_{\downarrow}}{N} = \alpha e^{\mu B/kT}$$

where α is a constant, and N is the total number of atoms per unit volume. The fraction of the atoms whose dipole

moment $\vec{\mu}$ is antiparallel to \vec{B} , $\frac{N_{\downarrow}}{N}$, will be

$$\frac{N_{\uparrow}}{N} = \alpha e^{-\mu B/kT},$$

As

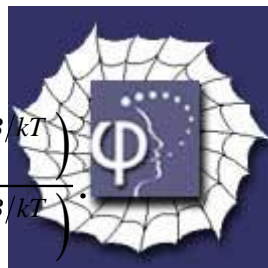
$$\frac{N_{\uparrow}}{N} + \frac{N_{\downarrow}}{N} = 1,$$

we note that

$$\alpha = \frac{1}{\left(e^{\mu B/kT} + e^{-\mu B/kT}\right)}.$$

The magnetization of the material, which is defined as the magnetic moment per unit volume, will therefore be given by the expression

$$\begin{aligned} M &= \mu N_{\uparrow} - \mu N_{\downarrow} \\ &= \mu N \frac{\left(e^{\mu B/kT} - e^{-\mu B/kT}\right)}{\left(e^{\mu B/kT} + e^{-\mu B/kT}\right)}. \end{aligned}$$



Let us call

$$\frac{\mu B}{kT} = x,$$

we thus find that

$$M = \mu N \tanh x.$$

(b)

We will consider two limiting cases, one when $x = 1$. In this limit as

The expression for magnetization simplifies to

$$M = \mu N x = \frac{\mu^2 B N}{kT}.$$

(c)

When $x \gg 1$,

$\tanh x \approx 1$,

and

$M \approx N\mu$.

