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<u>Problem 37.24 (RHK)</u>

Consider a solid containing N atoms per unit volume, each atom having a magnetic dipole moment $\vec{\mu}$. *Suppose the direction of* $\vec{\mu}$ *can be only parallel or antiparallel to an externally applied magnetic field* $\vec{B}$ (*this will be the case if* $\vec{\mu}$ *is due to the spin of a single electron*). *According to statistical mechanics, it can be shown that the probability of an atom being in a state with energy U is proportional to* $e^{-U/kT}$ *where T is the temperature and k is the Boltzmann's constant. Thus since* $U = -\vec{\mu}.\vec{B}$, *the fraction of atom whose dipole moment is parallel to* $\vec{B}$ *is proportional to* $e^{\mu B/kT}$ *and the fraction of atoms whose dipole moment is antiparallel to* $\vec{B}$ *is proportional to* $e^{-\mu B/kT}$. (a) *We have to show that the magnetization of this solid is* $M = N\mu \tanh\left(\mu B/kT\right)$. (b) *We have to show that* (a) *reduces to* $M = N\mu^2 B/kT$ *for* $\mu B \ll kT$. (c) *We have to show that* (a) *reduces to* $M = N\mu$ *for* $\mu B \gg kT$.

Solution:

Consider a solid containing N atoms per unit volume, each atom having a magnetic dipole moment $\vec{\mu}$. We suppose the direction of $\vec{\mu}$ can be only parallel or antiparallel to an externally applied magnetic field $\vec{B}$ (this will be the case if $\vec{\mu}$ is due to the spin of a single electron). According to statistical mechanics, it can be shown that the probability of an atom being in a state with energy U is proportional to $e^{-U/kT}$ where T is the temperature and k is the Boltzmann's constant.

As

$$U = -\vec{\mu}.\vec{B},$$

the fraction of the atoms whose dipole moment is parallel to $\vec{B}$, $\frac{N_{\uparrow}}{N}$,

$$\frac{N_{\downarrow}}{N} = \alpha e^{\mu B/kT}$$

where α is a constant, and N is the total number of atoms per unit volume. The fraction of the atoms whose dipole moment $\vec{\mu}$ is antiparallel to $\vec{B}$, $\frac{N_{\downarrow}}{N}$, will be

$$\frac{N_{\uparrow}}{N} = \alpha e^{-\mu B/kT},.$$

As

$$\frac{N_\uparrow}{N} + \frac{N_\downarrow}{N} = 1,$$

we note that

$$\alpha = \frac{1}{\left(e^{\mu B/kT} + e^{-\mu B/kT}\right)}.$$

The magnetization of the material, which is defined as the magnetic moment per unit volume, will therefore be given by the expression

$$M = \mu N_\uparrow - \mu N_\downarrow$$

$$= \mu N \frac{\left(e^{\mu B/kT} - e^{-\mu B/kT}\right)}{\left(e^{\mu B/kT} + e^{-\mu B/kT}\right)}.$$

Let us call

$$\frac{\mu B}{kT} = x,$$

we thus find that

$$M = \mu N \tanh x.$$

(b)

We will consider two limiting cases, one when

$x =$ 1. In this limit as

The expression for magnetization simplifies to

$$M = \mu N x = \frac{\mu^2 B N}{kT}.$$

(c)

When x ? 1,

$\tanh x$; 1,

and

M ; $N\mu$.