

515.

Problem 37.30 (RHK)

Using the results the problem 514 (Problem 37.29 (RHK)), we have to predict the value of the Earth's magnetic field (magnitude and inclination) at (a) the magnetic equator; (b) a point at magnetic latitude 60° ; and (c) the north magnetic pole.

Solution:

The magnetic field of the Earth can be approximated as a dipole magnetic field, with horizontal and vertical components, at a point a distance r from the Earth's centre, given by

$$B_h = \frac{\mu_0 \mu}{4\pi r^3} \cos L_m, \quad B_v = \frac{\mu_0 \mu}{2\pi r^3} \sin L_m,$$

where L_m is the magnetic latitude (latitude measured from the magnetic equator toward the north or south magnetic pole). The magnetic dipole moment is $8.0 \times 10^{22} \text{ A m}^2$. The radius of the Earth is $R_E = 6.37 \times 10^6 \text{ m}$.

(a)

The Earth's magnetic field at the magnetic equator will be

$$B_h = \frac{10^{-7} \times 8.0 \times 10^{22}}{(6.37 \times 10^6)^3} \text{ T} = 0.031 \times 10^{-3} \text{ T} = 31 \mu\text{T},$$

and

$$B_v = 0.$$

The inclination of the Earth's magnetic field at the magnetic equator will be zero.



(b)

The horizontal and the vertical components of the Earth's magnetic field at a point on its surface at the magnetic latitude 60° will be

$$B_h = \frac{\mu_0 \mu}{4\pi r^3} \cos 60^\circ = 15.5 \mu\text{T},$$

and

$$B_v = \frac{\mu_0 \mu}{2\pi r^3} \sin 60^\circ = 53.6 \mu\text{T}.$$

The magnitude of the Earth's magnetic field at the magnetic latitude 60° will be

$$B = (15.5^2 + 53.6^2)^{1/2} \mu\text{T} = 55.8 \mu\text{T}.$$

The angle of inclination is

$$\phi_i = \tan^{-1}\left(\frac{53.6}{15.5}\right) = 73.9^\circ.$$

(c)

The Earth's magnetic field at the magnetic north pole will be

$$B_v = 61.9 \mu\text{T},$$

and

$$B_h = 0.$$

The inclination of the Earth's magnetic field at the magnetic north pole will be

$$\phi_i = 90^\circ.$$

