514.

<u>Problem 37.29 (RHK)</u>

The magnetic field of the Earth can be approximated as a dipole magnetic field, with horizontal and vertical components, at a point a distance r from the Earth's centre, given by

$$B_h = \frac{\mu_0 \mu}{4\pi r^3} \cos L_m \ , \quad B_v = \frac{\mu_0 \mu}{2\pi r^3} \sin L_m \ ,$$

where L_m *is the magnetic latitude* (*latitude measured from the magnetic equator toward the north or south magnetic pole*). *The magnetic dipole moment is* $8.0 \times 10^{22}\ \text{A m}^2$. (a) *We have to show that the strength at latitude* L_m *is given by*

$$B = \frac{\mu_0 \mu}{4\pi r^3} \sqrt{1 + 3\sin^2 L_m}.$$

(b) *We have to show that the inclination* ϕ_i *of the magnetic field is related to the magnetic latitude* L_m *by*

$\tan\phi_i = 2\tan L_m$.

Solution:

The magnetic field of the Earth can be approximated as a dipole magnetic field, with horizontal and vertical components, at a point a distance r from the Earth's centre, given by

$$B_h = \frac{\mu_0 \mu}{4\pi r^3}\cos L_m \ , \quad B_v = \frac{\mu_0 \mu}{2\pi r^3}\sin L_m \ ,$$

where L_m is the magnetic latitude (latitude measured from the magnetic equator toward the north or south magnetic pole).

The strength of the magnetic field at the latitude L_m will be given by

$$B = \left(B_h{}^2 + B_v{}^2\right)^{1/2} = \frac{\mu_0 \mu}{4\pi r^3}\left(\cos^2 L_m + 4\sin^2 L_m\right)^{1/2}$$

$$= \frac{\mu_0 \mu}{4\pi r^3}\left(1 + 3\sin^2 L_m\right)^{1/2} .$$

The inclination ϕ_i of the magnetic field is defined as

$$\tan\phi_i = \frac{B_v}{B_h}.$$

We note that

$$\tan\phi_i = 2\tan L_m \ .$$