**513.**

**Problem 37.28 (RHK)**

*The magnetic dipole moment of the Earth is* $8.0\times10^{22}$ J T$^{-1}$. (a) *Assuming that the origin of this magnetism was a magnetised iron sphere at the centre of the Earth, we have to estimate its radius*. (b) *We have to find the fraction of the volume of the Earth that this sphere would occupy. The density of the Earth's inner core is* 14 g cm$^{-3}$. *The magnetic dipole moment of an iron atom is* $2.1\times10^{-23}$ J T$^{-1}$.

**Solution:**

The magnetic dipole moment of the Earth is $8.0\times10^{22}$ J T$^{-1}$.

A model for explaining the magnetic dipole moment of the Earth is to assume that there is a magnetised iron sphere at the centre of the Earth. Let the radius of this sphere be *R*. The magnetic dipole moment of an iron atom is $2.1\times10^{-23}$ J T$^{-1}$ and the density of the Earth's inner core is 14 g cm$^{-3}$.

The number density of iron atoms will therefore be

$$n = \rho \frac{N_A}{m}.$$

Here $\rho$ is 14 g cm$^{-3}$, $N_A$ is the Avogadro constant, and $m$ is the molar mass of iron.

Therefore,

$$n = 14 \times 10^3 \times \frac{6.02 \times 10^{23}}{0.0559} \text{ atoms per m}^3 = 1.507 \times 10^{29} \text{ atoms m}^{-3}.$$

The radius $R$ of the magnetised iron sphere will therefore be given by the relation

$$\frac{4\pi R^3}{3} \times 1.507 \times 10^{29} \times 2.1 \times 10^{-23} = 8.0 \times 10^{22},$$

or

$$R = \left( \frac{3 \times 8.0 \times 10^{22}}{4\pi \times 1.507 \times 2.1 \times 10^{6}} \right)^{1/3} \text{ m} = 181.9 \text{ km}.$$

The fraction of the volume of the Earth that this sphere would occupy would be

$$f = \left( \frac{R}{R_E} \right)^3 = \left( \frac{181.9}{6370} \right)^3 = 2.33 \times 10^{-5}.$$