510.

Problem 37.25 (RHK)

Consider an atom in which an electron moves in circular orbit with radius r and angular frequency ω_0. *A magnetic field is applied perpendicular to the plane of the orbit. As a result of the magnetic force, the electron circulates in an orbit with the same radius r but with a new frequency* $\omega = \omega_0 + \Delta\omega$. (a) *We have to show that when the field is applied the change in the centripetal acceleration of the electron is* $2r\omega_0\Delta\omega$. (b) *Assuming that the change in centripetal acceleration is entirely due to the magnetic force, we have to show that* $\Delta\omega = \pm eB/2m$.

Solution:

Let the speed of an electron of an atom in its circular orbit be v. Let an external magnetic field be applied to the atom such that it is perpendicular to the orbital plane of the electron in the atom. The electron could be orbiting in clockwise or the counter-clockwise directions as viewed from top of the orbit. The Lorentz force on the electron due to the external magnetic field will add to the centripetal force or reduce it depending on whether the electron is orbiting in the counter-clockwise or clockwise directions (charge of an electron is negative). We assume that the radius of the orbit does not change due to the Lorentz force and the effect is the change in the orbital frequency.

The Lorentz force on the electron will be $\pm evB$, depending on the relative direction between the magnetic field and the velocity of the electron.

Let the orbital angular frequency of the electron in the atom before the external magnetic field is applied be ω_0.

As the radius of the circular orbit is taken to be *r*, the relation between the centripetal force and the centripetal acceleration is

$$m\omega_0{}^2 r = F_c.$$

When external magnetic field is applied, as discussed above, the changed centripetal force will be

$$F_c \pm evB.$$

Let the changed angular frequency of the electron in the atom, assuming that its radius does not change, be

$\omega_0 + \Delta\omega$. The equation of the circular motion will now be

$$m(\omega_0 + \Delta\omega)^2 r = F_c \pm evB.$$

Assuming that $\Delta\omega/\omega_0 = 1$,

we approximate the equation of circular motion as

$$m\omega_0{}^2 r + 2m\omega_0\Delta\omega r = F_c \pm evB,$$

or

$$\Delta\omega = \pm\frac{evB}{2mr\omega_0}\ ;\ \pm\frac{eB}{2m}.$$

And note that the change in the centripetal acceleration is $2r\omega_0\Delta\omega$.