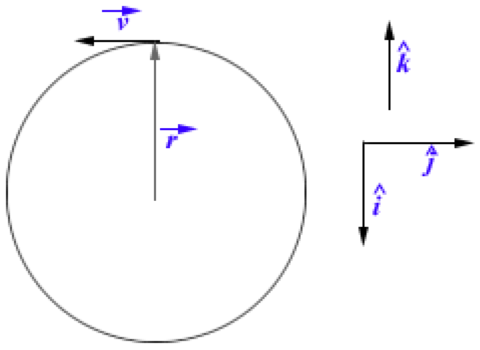
509.

<u>Problem 37.14 (RHK)</u>

An electron with kinetic energy K_e *travels in a circular path that is perpendicular to a uniform magnetic field, subject only to the force of the field. We have to show* (a) *that the magnetic dipole moment due to its orbital motion has magnitude* $\mu = K_e / B$ *and that it is in the direction opposite to that of* $\vec{B}$. (b) *We have to find the magnetic dipole moment of a positive ion with kinetic energy* K_i *under the same circumstances*. (c) *An ionised gas consists of* 5.28×10^{21} *electrons per* m^3 *and the same number of ions per* m^3. *We may take the average electron kinetic energy to be* 6.21×10^{-20} J *and the average ion kinetic energy to be* 7.58×10^{-21} J. *We have to calculate the magnetization of the gas for magnetic field of* 1.18 T.

Solution:

(a)



Let the magnetic field be in the positive z-direction, which is perpendicular to the plane of the page and is coming out of it.

$\vec{B} = B\hat{k}$.

The speed of an electron of an electron of an electron of mass m in terms of its kinetic energy K_e is given by

$$\frac{1}{2}mv^2 = K_e,$$

or

$$v = \sqrt{\frac{2K_e}{m}}.$$

As the charge of electrons is $-|e|$ and the Lorentz force in a magnetic field $\vec{B}$ will be

$$\vec{F} = -|e|\vec{v} \times \vec{B} = -|e|\vec{v} \times B\hat{k} = \hat{i}F,$$

$$\vec{v} = -v\hat{j}.$$

And

$$F = |e|vB.$$

Under the action of the magnetic field $\vec{B}$ electron is moving in a circular orbit of radius r. Therefore,

$$\frac{mv^2}{r} = |e|vB,$$

and

$$\frac{v}{r} = \frac{|e|B}{m}.$$

A uniformly circulating charge e is effectively a current

$$i = \frac{|e|}{T} = \frac{|e|\omega}{2\pi} = \frac{|e|v}{2\pi r}.$$

Current flowing in a closed loop enclosing area A is equivalent to a magnetic dipole of moment $\mu = iA$.

$$\therefore \ \mu = iA = \frac{|e|\omega}{2\pi}\pi r^2 = \frac{|e|vr}{2} = \frac{|e|v}{2} \times \frac{mv}{|e|B} = \frac{mv^2}{2B} = \frac{K_e}{B}.$$

As the electron has negative charge and is moving in counter-clockwise direction, the current flow will be in the clockwise direction, and therefore the direction of the orbital magnetic dipole moment will be $-\hat{k}$. That is

$$\vec{\mu} = -\frac{K_e}{B}\hat{k}.$$

We note that the orbital magnetic dipole moment of an electron in a magnetic field $\vec{B}$ is in the direction opposite to that of $\vec{B}$.

(b)

For the case of a positive ion of kinetic energy K_i the orbital magnetic dipole moment will be parallel to $\vec{B}$ and its magnitude will be K_i/B.

(c)

We have to find the magnetization of the gas consisting of electrons and ions of equal number density

$n = 5.21 \times 10^{21}$ m^{-3}.

We are given that

$K_e = 6.21 \times 10^{-20}$ J,

and

$K_i = 7.58 \times 10^{-21}$ J.

The magnetic field is

$B = 1.18$ T.

Therefore, the magnetization of the gas, assuming that all electrons and ions are moving in circular paths perpendicular to an external magnetic field, which is net dipole moment per unit volume, will be

$$M = \frac{\left(6.21 - 0.758\right) \times 10^{-20} \times 5.21 \times 10^{21}}{1.18} \text{ A m}^{-1}$$

$$= 2.24 \text{ A m}^{-1}.$$