508.

Problem 37.10 (RHK)

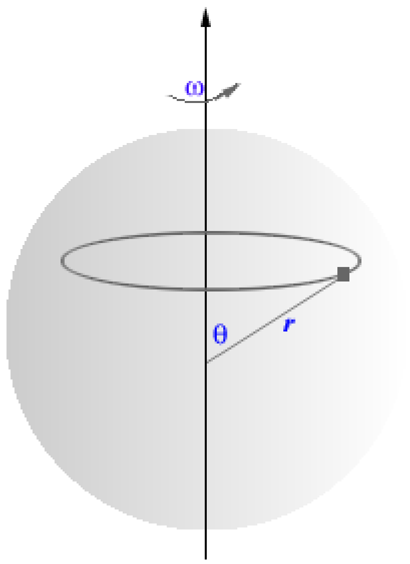
Assume that the electron is a small sphere of radius R, its charge and mass being spread uniformly throughout its volume. Such an electron has a "spin" angular momentum L and a magnetic moment μ. *We have to show that* $e/m = 2\mu/L$. *Is this prediction in agreement with experiment?*

Solution:

We assume that the electron is a small sphere of radius R, its charge and mass being spread uniformly throughout its volume. Under this assumption the charge density inside the electron will be

$$\rho = \frac{e}{\frac{4\pi}{3}R^3} = \frac{3e}{4\pi R^3}.$$

In this model of the electron uniformly charged sphere of radius R containing total charge e is rotating with angular speed ω. We will calculate the magnetic moment due to the rotating charged sphere.



We use spherical polar coordinate r, θ, φ. The magnetic moment due to rotating charge contained in the ring as shown in the diagram will be

$$d\mu(r) = \frac{1}{2} q\omega \left(r \sin\theta \right)^2,$$

where q the total charge contained in the ring shown is $q = 2\pi r^2 \sin\theta \rho d\theta dr$. As the magnetic moments due to each element of rotating charges are parallel, their contributions add. Therefore, the magnetic moment due to charge contained in the rotating sphere of radius R will be

$$\mu = \int_0^R dr \int_0^\pi \pi r^4 \rho\omega \sin^3\theta d\theta.$$

Note that the integral

$$\int_0^\pi \sin^3\theta d\theta = \frac{4}{3}.$$

Therefore,

$$\mu = \int_0^R dr \int_0^\pi \pi r^4 \rho\omega \sin^3\theta d\theta = \frac{4\pi\rho\omega}{3}\int_0^R r^4 dr = \frac{4\pi\rho\omega R^5}{15}.$$

Substituting the value of ρ, we find for the magnetic moment of a sphere of radius *R* containing charge *e* and rotating with angular speed ω the expression

$$\mu = \frac{e\omega R^2}{5}.$$

We recall that the rotational inertia of a sphere of radius *R* containing mass total *m* which is uniformly distributed is

$$I = \frac{2}{5}mR^2.$$

Therefore, the angular momentum of the “spinning” sphere of radius R rotating with angular speed ω will be

$$L = I\omega = \frac{2mR^2\omega}{5}.$$

We thus find that

$$\frac{\mu}{L} = \frac{e\omega R^2}{5} \times \frac{5}{2m\omega R^2} = \frac{e}{2m}.$$

The experimental value of magnetic moment of an electron is

$$\mu_e = \frac{e}{m} \times \frac{\hbar}{2},$$

where

$$\hbar = \frac{h}{2\pi},$$

$h(\text{Planck constant}) = 6.63 \times 10^{-34}$ J s.

The spin of an electron

$$s = \frac{\hbar}{2}.$$

Therefore,

$$\frac{\mu_e}{s} = \frac{e}{m}.$$

Therefore, the result of our model is in disagreement with experiment. Our model of electron is too mechanistic and is not in the spirit of quantum mechanics.