507.

Problem 37.9 (RHK)

A charge q is distributed uniformly around a thin ring of radius r. The ring is rotating about an axis through the centre and perpendicular to its plane at an angular speed ω. *We have to show that the magnetic moment due to the rotating charge is*

$$\mu = \tfrac{1}{2} q\omega r^2.$$

(b) *If L is the angular momentum of the ring, we have to show that* $\mu/L = q/2m$.

Solution:

We will first calculate the current in a ring of radius r containing charge q which is uniformly distributed when the ring is rotating about an axis through the centre and perpendicular to its plane at an angular speed ω. Current at a location is defined as the charge flowing per second. An amount of charge q flows through at any point on the ring in time

$$T = \frac{2\pi r}{r\omega} = \frac{2\pi}{\omega}.$$

Therefore, the current i in the rotating ring is

$$i = \frac{q}{T} = \frac{q\omega}{2\pi}.$$

As the current i is flowing in a closed loop of area πr^2 the magnetic moment will be

$$\mu = \pi r^2 i = \pi r^2 \frac{\omega q}{2\pi} = \frac{1}{2}\omega q r^2.$$

(b)

Let m be the mass of the charge q contained in the ring.

As the ring is rotating with angular speed ω, the angular momentum of the rotating charge will be

$$L = mr^2\omega.$$

$$\therefore \quad \frac{\mu}{L} = \frac{\frac{1}{2}\omega q r^2}{mr^2\omega} = \frac{q}{2m}.$$