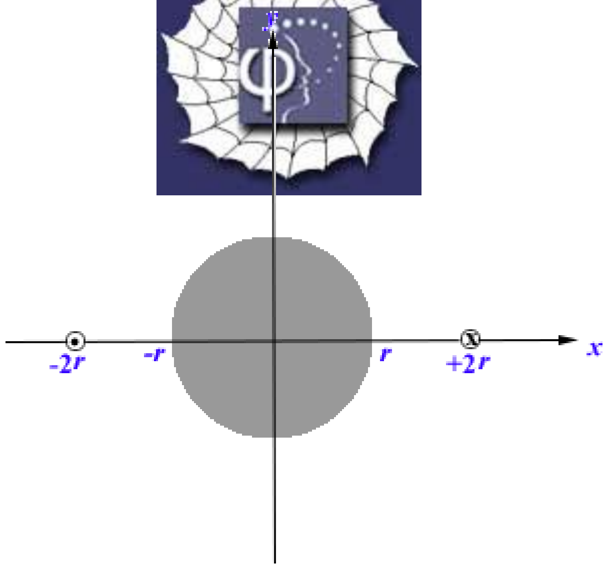
505.

<u>Problem 37.5 (RHK)</u>

Two wires, parallel to the z axis and a distance 4r apart, carry equal currents i in opposite directions, as shown in the figure. A circular cylinder of radius r and length L has its axis on the z axis midway between the wires. Using Gauss' law for magnetism we have to calculate the net out ward magnetic flux through the half of the cylindrical surface above the x axis.



<u>Solution:</u>

Two wires, parallel to the z axis and a distance 4r apart, carry equal currents i in opposite directions, as shown in the figure. A circular cylinder of radius r and length L has its axis on the z axis midway between the wires.

The Gauss' law for the magnetic field states that the net outward flux of magnetic field for any closed Gaussian surface *S* is zero. That is

$$\iint_S \vec{B}.d\vec{s} = 0 \ .$$

For calculating the net outward magnetic flux through half of the cylindrical surface above the *x*-axis we consider a Gaussian surface *S* enclosed by the *xz*-plane within the cylinder dividing the cylinder vertically and the half of the cylindrical surface above the *x*-axis (*A*). As the magnetic field due to the current carrying wires which are parallel to the z-axis will be circular, the contribution to the flux from the top and bottom semicircular surfaces of *S* will be zero.

Therefore,

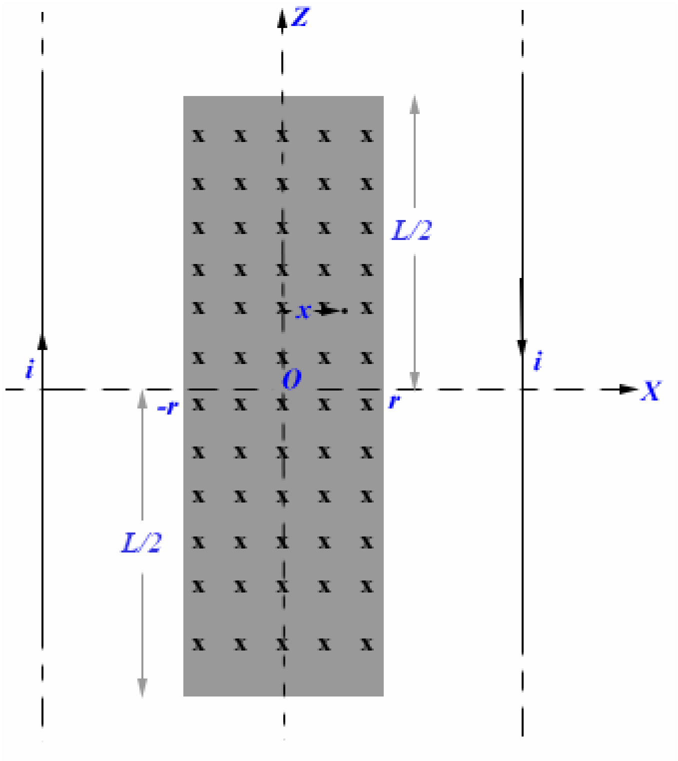
$$\iint_{\left(\substack{\text{half of the cylidrical}\\ \text{surface above the xaxis}}\right)} \vec{B}.d\vec{s} + \iint_{\text{A}} \vec{B}.d\vec{s} = 0,$$

Or

$$\iint_{\left(\substack{\text{half of the cylidrical}\\ \text{surface above the xaxis}}\right)} \vec{B}.d\vec{s} = -\iint_{\text{A}} \vec{B}.d\vec{s} \ .$$

We will next calculate

$$\iint_{\text{A}} \vec{B}.d\vec{s} \ .$$



The magnetic field due to the two long wires carrying current i in opposite directions, as shown in the figure, at a distance x from the z axis will be

$$B(x) = \frac{\mu_0 i}{2\pi(2r - x)} + \frac{\mu_0 i}{2\pi(2r + x)}.$$

Therefore,

$$\iint_{A} \vec{B}.d\vec{s} = 2L\int_0^r (-)\left(\frac{\mu_0 i}{2\pi}\right)\left(\frac{1}{2r - x} + \frac{1}{2r + x}\right)dx$$

$$= -\frac{\mu_0 iL}{\pi}\left(\ln\left(\frac{2r}{r}\right) + \ln\left(\frac{3r}{2r}\right)\right) = -\frac{\mu_0 iL}{\pi}\ln 3.$$

We thus find that

$$\iint_{\left(\substack{\text{half of the cylidrical} \\ \text{surface above the xaxis}}\right)} \vec{B}.d\vec{s} = -\iint_{A} \vec{B}.d\vec{s}$$

$$= \frac{\mu_0 iL}{\pi}\ln 3.$$