504.

<u>Problem 36.47 (RHK)</u>

In a certain betatron, the radius of the electron orbit is 32 cm *and the magnetic field at the orbit is given by* $B_{orb} = 0.28\sin 120\pi t$, *where t is in seconds and* B_{orb} *is in tesla. In the betatron, the average value* B_{av} *of the field enclosed by the electron orbit is equal to twice the value* B_{orb} *at the electron orbit.* (a) *We have to calculate the induced electric field felt by the electrons at* $t = 0$. *We have to find the acceleration of the electrons at this instant. We may ignore the relativistic effects.*

<u>Solution:</u>

In the betatron the magnetic field at the orbit is

$B_{orb} = 0.28\sin 120\pi t$.

and as the average value B_{av} of the field enclosed by the electron orbit is equal to twice the value B_{orb} at the electron orbit, we have

$B_{av} = 2B_{orb} = 0.56\sin 120\pi t$ T.

As the radius of the electron orbit is 32 cm, the flux enclosed by the electron orbit will be

$$\Phi(t) = \pi r^2 B_{av} = \pi \times (0.32)^2 \times 0.56 \sin 120\pi t \text{ T m}^2$$

and

$$\frac{d\Phi(t)}{dt} = \pi \times (0.32)^2 \times 0.56 \times 120\pi \cos 120\pi t \text{ V},$$

and

$$\left.\frac{d\Phi(t)}{dt}\right|_{t=0} = \pi \times (0.32)^2 \times 0.56 \times 120\pi \text{ V} = 67.91 \text{ V}.$$

By applying the Faraday's law, we find that the induced electric field at the orbit will be

$$2\pi r E = 67.91 \text{ V},$$

and

$$E = \frac{67.91}{2\pi \times 0.32} \text{ V m}^{-1} = 33.77 \text{ V m}^{-1}.$$

The acceleration of the electrons at this instant will therefore be

$$a = \frac{eE}{m} = \frac{33.77 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} \text{ m s}^{-2} = 5.93 \times 10^{12} \text{ m s}^{-2}.$$