503.

Problem 36.44 (RHK)

Early in 1981 the Francis bitter National Magnet laboratory at M.I.T. commenced operation of a 3.3-cm diameter cylindrical magnet, which produces a 30-T field, then the world's largest steady-state field. The field can be varied sinusoidally between the limits of 29.6 T and 30.0 T at a frequency of 15 Hz. We have to find the maximum value of the induced electric field at a radial distance of 1.6 cm from the axis.



Solution:

Because the magnetic field is cylindrical, the induced electric field that will appear with the change in magnetic field will have cylindrical symmetry. That is it will have constant magnitude at the same distance from the axis. The field will be tangential as shown in the figure. We apply the Faraday's law in calculating the induced electric field at a radial distance of 1.6 cm from the axis.

$$\mathbf{\tilde{N}}^{\mathbf{r}}_{E}.ds^{\mathbf{r}} = -\frac{d\Phi}{dt}.$$

We note that

$$\mathbf{\tilde{N}}^{\mathbf{L}}.ds^{\mathbf{r}}=2\pi rE.$$

We estimate the maximum rate of change of flux by approximating that the change in magnetic field from 29.6 T to 30.0 T takes place in $\Delta t = \frac{1}{2} \times \frac{1}{2} \text{ s} = \frac{1}{2} \text{ s}.$

$$\therefore \frac{\Delta \Phi}{\Delta t} = \pi r^2 (30.0 - 29.6) \times 30 \text{ V}.$$

And

$$E_{\text{max}} = \frac{\pi r^2 (30.0 - 29.6) \times 30}{2\pi r} \text{ V m}^{-1} = \frac{1}{2} \times 1.6 \times 10^{-2} \times 0.4 \times 30 \text{ V m}^{-1}$$
$$= 9.6 \times 10^{-2} \text{ V m}^{-1}.$$