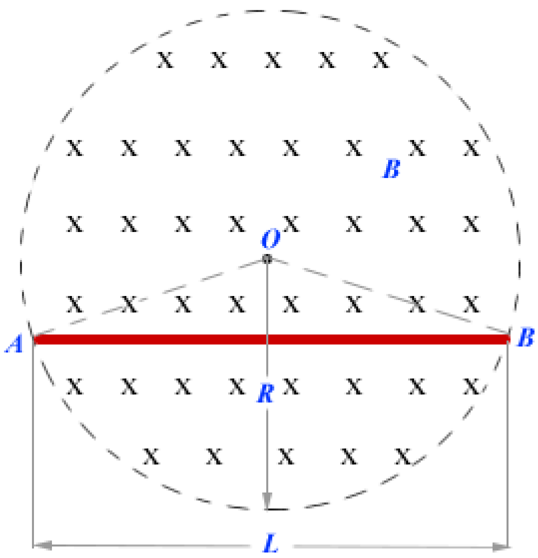
502.

<u>Problem 36.45 (RHK)</u>

A uniform magnetic field $\vec{B}$ fills a cylindrical volume of radius R. A metal rod of length L is placed as shown in the figure. If B is changing at the rate dB/dt, we have to show that the emf that is produced by the changing magnetic field and that acts between the ends of the rod is given by

$$\mathcal{E} = \frac{dB}{dt}\frac{L}{2}\sqrt{R^2 - \left(\frac{L}{2}\right)^2}\ .$$



Solution:

Because of the cylindrical symmetry and the direction of the magnetic field as shown in the figure, the electric field that will appear because of the change in flux due to change in the magnetic field will be circular. The induced electric field will not have radial component.

For computing the induced emf across the ends of the rod, which is placed in the magnetic field as shown in the figure, we join its ends *A* and *B* with the point *O* on the axis as shown.

We compute $\oint \vec{E}.d\vec{s}$ along the closed contour *AOBA*. As the induced electric field $\vec{E}$ does not have radial component, the contour integral

$$\oint \vec{E}.d\vec{s} = \int_B^A \vec{E}.d\vec{s} = -\left(V_A - V_B\right) = -\mathcal{E}..$$

By Faraday's law of induction,

$$\oint \vec{E}.d\vec{s} = -\frac{d\Phi}{dt} \ ,$$

where Φ is the flux enclosed.

The flux enclosed by the loop *AOBA* will be B times the area of the triangle *AOB*. It, therefore, will be

$$\Phi = B\frac{L}{2}\left(R^2 - \left(\frac{L}{2}\right)^2\right)^{1/2},$$

and

$$\frac{d\Phi}{dt} = \frac{dB}{dt}\frac{L}{2}\left(R^2 - \left(\frac{L}{2}\right)^2\right)^{1/2}.$$

Therefore, the emf produced by the changing magnetic field across the ends of the rod will be

$$\mathrm{E} = \frac{dB}{dt}\frac{L}{2}\sqrt{R^2 - \left(\frac{L}{2}\right)^2}\ .$$