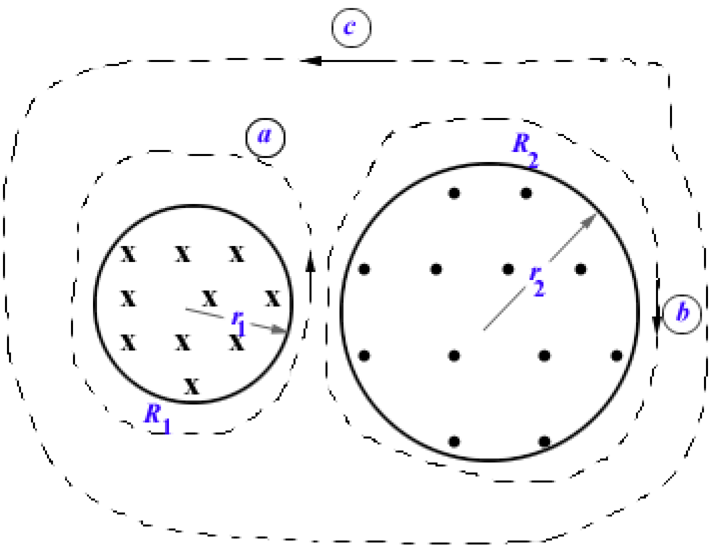
501.

Problem 36.41 (RHK)

In the figure two circular regions R_1 *and* R_2 *have been shown. Their radii are* $r_1 = 21.2\ \text{cm}$ *and* $r_2 = 32.3\ \text{cm}$, *respectively. In* R_1 *there is a uniform magnetic field* $B_1 = 48.6\ \text{mT}$ *into the page and in* R_2 *there is a uniform magnetic field* $B_2 = 77.2\ \text{mT}$ *out of the page (ignoring any fringing of these fields). Both fields are decreasing at the rate* $8.50\ \text{mT s}^{-1}$. *We have to calculate the integral* $\oint \vec{E}.d\vec{s}$ *for each of the three indicated paths.*



Solution:

According to Faraday's law of induction

$$\oint \vec{E}.d\vec{s} = -\frac{d\Phi}{dt} ,$$

where Φ is the flux enclosed by the closed curve.

(a)

We will calculate $\oint \vec{E}.d\vec{s}$ first for the path (a). It is given that the magnetic field is decreasing in both regions R_1 and R_2 at the uniform rate of 8.50 mT s^{-1}. We note that the uniform magnetic field in the region R_1 is into the plane of the figure. Therefore, induced current will flow in the clockwise direction to compensate for the decrease in flux in the region R_1. But the contour integral is being calculated in the counter-clockwise sense; therefore, its value will be negative. Therefore,

$$\oint \vec{E}.d\vec{s} = -\pi r_1^2 \times 8.50 \times 10^{-3} \text{ V} = -\pi (0.212)^2 \times 8.50 \times 10^{-3} \text{ V}$$

$$= -1.20 \text{ mV}.$$

(b)

We will calculate $\oint \vec{E}\cdot d\vec{s}$ first for the path (b). It is given that the magnetic field is decreasing in both regions R_1 and R_2 at the uniform rate of 8.50 mT s^{-1}. We note that the uniform magnetic field in the region R_2 is coming out of the plane of the figure. Therefore, induced current will flow in the counter-clockwise direction to compensate for the decrease in flux in the region R_2. But the contour integral is being calculated in the clockwise sense; therefore, its value will be negative. Therefore,

$$\oint \vec{E}\cdot d\vec{s} = -\pi r_2^2 \times 8.50\times10^{-3}\text{ V} = -\pi\left(0.323\right)^2 \times 8.50\times10^{-3}\text{ V}$$

$$= -2.79\text{ mV}.$$

(c)

We will calculate $\oint \vec{E}\cdot d\vec{s}$ first for the path (c). It is given that the magnetic field is decreasing in both regions R_1 and R_2 at the uniform rate of 8.50 mT s^{-1}. As r_2 is greater than r_1 the net decrease of flux in the region enclosed by the contour (c) will be

$$\pi\left(r_2^2 - r_1^2\right)\frac{dB}{dt} = \left(2.79 - 1.20\right)\text{ mV}.$$

Therefore, induced current will flow in the counter-clockwise direction to compensate for the decrease in flux enclosed. The sense of the contour integral is that of the flow of the induced current; therefore, its value will be negative. Therefore,

$$\oint \vec{E}.d\vec{s} = 1.59 \text{ mV}.$$