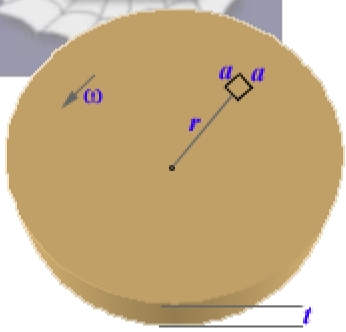
500.

Problem 36.39 (RHK)

An electromagnetic "eddy current" brake consists of a disk of conductivity σ and thickness t rotating about an axis through its centre with a magnetic field $\vec{B}$ applied perpendicular to the plane of the disk over a small area a^2 (see figure). If the area a^2 is at a distance r from the axis, we have to find an approximate expression for the torque tending to slow down the disk at the instant its angular velocity equals ω.



Solution:

As the disk is rotating with angular speed ω, the element of area a^2 at a distance r from the centre of disk will have a tangential velocity of magnitude $r\omega$. It is mentioned that a magnetic field perpendicular to the plane of the disk is localised over the area a^2. The element of area a^2 will cross the magnetic field in time interval

$$\Delta t = \frac{a}{r\omega}.$$

There will be a change in flux in the element of area a^2 and an emf will be generated. The magnitude of the emf will be

$$\mathrm{E} = \frac{a^2 B}{a/r\omega} = aB\omega r.$$

As the charge carriers contained inside the element of area a^2 and thickness t of the conducting material of the disk will be moving with speed v, localised eddy currents will arise in this element of area. The electrical resistance offered to the eddy currents will be approximately

$$R = \frac{a}{\sigma a t}.$$

The rate of internal energy dissipation due to the eddy currents in the element of area a^2 will be

$$P = \frac{\mathrm{E}^2}{R} = \frac{(aB\omega r)^2}{1/\sigma t} = (aBr)^2\,\omega^2 \sigma t.$$

As the angular speed of the disk at the instant when the element of area a^2 rotating with angular speed ω passes over the magnetic field, a torque N will arise, which can be obtained by equating $N\omega$ with the rate of internal energy dissipation P. We have

$$N\omega = \left(aBr\right)^2 \omega^2 \sigma t,$$

or

$$N = \left(aBr\right)^2 \omega\sigma t.$$